

The Missing Problems of Gersonides—A Critical Edition I

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Gersonides' *Maaseh Hoshev* (*The Art of Calculation*) is a major work known for its early use of rigorous combinatorial proofs and mathematical induction. There is a large section of problems at the end of the book, with the theme of proportions, which until now has remained unpublished. I present a critical edition of this material. I also uncover a previously unknown second edition of *Maaseh Hoshev*. The material is appropriate for creative pedagogy and provides economic details of the author's culture and environs. This article presents the first 15 problems and a subsequent article presents the rest. © 2000 Academic Press

מעשה חושב לרלב"ג הוא חיבור חשוב הידוע הודות לשימושו המוקדם בהוכחות קומבנטוריות מדויקות ובאינדוקציה מתמטית. בסוף הספר מופיע פרק גדול של שאלות שנושאן יחסים, שטרם פורסם. אני מציג הוצאה ביקורתית של פרק זה. החומר חושף מהדורה שנייה לא ידועה של מעשה חושב. החומר מתאים לפדגוגיה יצירתית ומספק פרטים כלכליים על אורח חייו וסביבתו של המחבר. במאמר זה מובאות חמש עשרה הבעיות הראשונות והיתר יופיעו במאמר נוסף. © 2000 Academic Press

Le *Maase Hoshev* (*L'Art du Calcul*) de Gersonide est un ouvrage majeur connu pour son usage précoce de preuves combinatoires rigoureuses et de l'induction mathématique. La fin de l'ouvrage comporte une section importante consacrée à une série de trente problèmes sur le thème des proportions qui était restée inédite jusqu'à ce jour et don't je présente pour la première fois une édition critique. Je révèle également l'existence d'une seconde édition de *Maase Hoshev*. Le matériau peut servir de base à des projets de pédagogie créative et offre des détails sur la culture et l'environnement de l'auteur. Cet article présente les quinze premiers problèmes de *Maase Hoshev*; un second article en présente les quinze autres. © 2000 Academic Press

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I. INTRODUCTION

A critical edition of a collection of heretofore unpublished problems by the noted medieval scientist, philosopher and mathematician, Levi ben Gershon, a.k.a. Gersonides, with translation and commentary, is presented. The publication of these problems coincides with my discovery of a second edition of *Maaseh Hoshev*, the book which contains these problems.

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The mathematical theme underlying all the problems is the use of proportions. The problems are often presented in a practical manner and thereby reflect and confirm historical prices, measures, exchange rates for different coins, wages, and types of goods and services in 14th century Provence. The problems also provide useful material for creative pedagogy.

The critical edition is presented in two articles. This first article includes the first 15 problems. The second article, to appear in a subsequent issue of this journal, includes Problems 16–21. These later problems contain a number of difficult sections and errors.

II. LEVI BEN GERSHON, A.K.A. GERSONIDES

Levi ben Gershon (1288–1344), rabbi, philosopher, astronomer, scientist, biblical commentator, and mathematician, was born in Provence (south France) and lived there all his life. Through his writings, he distinguished himself as one of the great medieval scientists and a major philosopher. He wrote more than a dozen books of commentary on the Old Testament, a major philosophical work called *MilHamot Adonai* (*Wars of God*), a book on logic, four treatises on mathematics, and a variety of other scientific and philosophical commentaries. *MilHamot Adonai* has a section on trigonometry and a long section on astronomy, including the invention of the Jacob's staff, a device to measure angles between heavenly bodies used for centuries by European sailors for navigation, a discussion of the *camera obscura*, and original theories on the motion of the moon and planets. A complete bibliography on Levi and his work can be found in [18].

There is little information about Levi's personal life except that he came from a family of scholars and learned men. Although the details of these relationships are not firmly established—for example, one can find no less than three different claims (Levi ben Abraham ben Hayim, Gershon ben Shlomo, and NaHmonides) for the identity of his grandfather—it is well known that his home of Provence in southern France was a place that supported productive Jewish scholarship in the 14th century.

Life for the Jews in 14th-century Provence was relatively good. The Pope, who was then at Avignon, protected them from oppression, which was not the case in the northern parts of France. There were 15,000 Jews among the two million general population of 14th-century Provence. Jews worked as moneylenders, physicians, craftsman, and merchants.

Levi was highly regarded by the Christian community as a scientist and mathematician. He is referred to by them as Leo Hebraus, Leo de Balneolis, or Maestro Leon. His book on trigonometry was dedicated to Pope Clement VI and his book *On Harmonic Numbers* was commissioned by Phillip of Vitry, Bishop of Meaux. Both of these works were translated from Hebrew to Latin in Levi's lifetime. Levi wrote only in Hebrew and read Hebrew translations of Latin and Arabic works, readily available to him.

Despite his originality and reputation, much of Levi's scientific and mathematical work did not heavily influence his successors. It is not clear to what extent he played a role in the transmission of Hellenistic mathematics from the Arab world to western Europe.

Levi's Mathematics

Levi's mathematics comprises four major works.

A commentary on Euclid was completed in the early 1320s, with ready access to Hebrew translations. Included is an attempt to prove the fifth postulate. The content is discussed in [23].

De Sinibus, Chordis et Arcubus, a treatise on trigonometry in *MilHamot Adonai*, was completed in 1342 and dedicated to the Pope. There is a translation of this work into German [6] and an expository article on its content [10].

De Numeris Harmonicis, completed in 1343, was commissioned by Phillip de Vitry, Bishop of Meaux, and immediately translated from Hebrew into Latin. Philip was a musicologist interested in numbers of the form $2^n 3^m$ called harmonic numbers. Levi proves that the only pairs of harmonic numbers that differ by one are (1, 2), (2, 3), (3, 4), and (8, 9). The book is relatively short and the original Hebrew is lost. The Latin version and commentary can be found in [5]. A summary of the contents can be found in [7].

Maaseh Hoshev, Levi's first and largest mathematics book dated 1321, contains the missing problems presented here. It is known for its early use of mathematical induction. Translations and work discussing its content include [5; 7; 16; 17; 20; 21; 26; 31; 32].

III. MAASEH HOSHEV (THE ART OF CALCULATION)

Maaseh Hoshev (The Art of Calculation), dated 1321, is Levi's first book on mathematics and his second book overall. (Levi's first book is on logic and is dated 1319 [24]). The title comes from the biblical book of Exodus, where the phrase is used in a number of different places to describe the type of work necessary in constructing the tabernacle. Although it literally means "A Work of Calculation," its true meaning is more subtle. It is translated below as "cunning work."

Exodus 29:2–3

And he made the ephod of gold, blue, purple, and scarlet, and fine twisted linen. And they did beat the gold into thin plates, and cut it into wires, to work it in the blue, and in the purple, and in the scarlet, and in the fine linen, with *cunning work*.

This *cunning work* is in contrast with other types of work in the construction of the tabernacle, which are referred to by the skill required, such as *maaseh rokem*, the work of an embroiderer, or *maaseh oreg*, the work of a weaver, or *maaseh harash*, the work of an engraver, or *maaseh avot*, the work of a braider. The intention is that *maaseh hoshev* denotes a skill that requires more than just technical craftsmanship. It is the skill of an architect, requiring thought, cunning, planning and calculation.

The title is also a play on words for theory and practice, *Maaseh* corresponding to practice and *Hoshev* corresponding to theory. Levi writes in his introduction to the book:

It is only with great difficulty that one can master the art of calculation, without knowledge of the underlying theory. However, with the knowledge of the underlying theory, mastery is easy ... and since this book deals with the practice and the theory, we call it *Maaseh Hoshev*.

Maaseh Hoshev is a major work in two parts. Part one is a collection of 68 theorems and proofs in Euclidean style about arithmetic, algebra, and combinatorics. Part two contains algorithms for calculation and is subdivided into six sections:

- a. Addition and Subtraction
- b. Multiplication
- c. Sums
- d. Combinatorics
- e. Division, Square Roots, Cube Roots
- f. Ratios and Proportions.

A large collection of problems appears at the end of Section f in part two. Not including this collection of problems, parts one and two are about the same size, and the problems are about a third the size of each part. The text and the problem section of part two often refer back to the theorems in part one. Levi lists Euclid, books 7–9, as prerequisite reading.

Lange's Critical Edition

Lange wrote a critical edition of *Maaseh Hoshev* in 1909 with an introduction, translation, and commentary in German [20]. It ignores the problem section completely and includes nothing about the second edition.

Lange had knowledge of only four mss. In his introduction, he mentions that his main source was the Vienna ms. because it is the most complete, and that he used the Munich 68 ms. to help distinguish between letters in the Vienna ms. that look similar. He adds that he never actually saw the Paris ms., but was able to learn some details about it through a friend. It should be noted that the Paris and Vienna mss. include all the missing problems, while Munich 36 has none, and Munich 68 contains just three. Also note that the Vienna and Munich 68 mss. represent the same family of mss.

Lange and the Missing Problems

As far as the problem section is concerned, Lange describing the Vienna ms., writes:

Wir mussten sie einer späteren Veröffentlichung vorbehalten, da es aus gewissen Gründen nicht möglich war, sie jetzt mit zu veröffentlichen.

[We had to leave the problems for a future publication; because of specific reasons, it was not possible to publish them here.]

Lange never published a subsequent paper on the missing problems. I speculate that this was because:

1. He was unable to work his way through the maze of Problem 17 with only one source.
2. There are a number of errors in other problems, which are hard to resolve, especially without being able to distinguish between the similar-looking letters present in the Vienna ms. He was unable to distinguish these letters because the Munich ms. he used for this purpose is missing most of the problem section.

Two Editions of Maaseh Hoshev

In the literature, *Maaseh Hoshev* is dated 1321; however, this information is incomplete. There are, in fact, two editions of *Maaseh Hoshev*. One is the basis of Lange's 1909 critical edition, and the other is dated one and one-half years later. The editions can be distinguished by their colophons which date the edition either as 1321 or 1322. When the colophon is missing, the edition can be identified by other distinguishing features.

The major difference between the two editions is the omission in the second edition of two problems, one on the solution of certain quadratic equations and the other, a very long problem, on the solution of certain simultaneous linear equations. The theorems in part one related to these problems are also omitted. There are other minor additions, changes, and omissions in both the theorems of part one and the algorithms and problems of part two. The theorems in part one of the second edition are renumbered due to these

changes, and these new numbers are used in part two of the second edition for referencing purposes.

Why a Second Edition?

Levi has no special preface, nor does he give any explanation for the second edition, but it is clear that the second edition is simply a reworking of the text for the ease of the reader. The differences in the second edition are primarily not in content but in organization and presentation. In the second edition, Problems 16 and 17 are omitted, and Problems 14 and 15 are completely rewritten. Problems 14 and 15 of the second edition present the same material as Problems 14 and 15 of the first edition but use techniques that are more easily generalized. Problem 16 deals with the solution of quadratic equations and is the only one of all 21 problems that does not use proportions. The problem does not fit in naturally with the rest of the section. Problem 17 also differs from the rest of the problems. It is about 10 times the length of any of the other problems and includes very long tedious proofs with difficult error-prone notation rendering them all but impossible to read. What's more, Problem 21 reviews the main points contained in Problem 17 in a much more condensed and simple fashion. Finally, the only real error in the whole book, that is, one which is clearly not the result of a careless or ignorant scribe and one which cannot be corrected or reconstructed, is contained in Problem 17.

A final and important difference between the two editions, is that the colophon of the first edition appears preceded by the words "The author writes:" and has a self-reference to Levi's age at the date given; while the colophon of the second edition has neither. It is Levi's consistent style to start all his work with the words "Levi says," thereby identifying himself as the author. It is possible that Levi left out the personal references in the colophon of the second edition for no special reason except that it was a second edition. However, it is also possible that he wanted to give credit to students who may have helped in the editing process. Glasner's work [13] implying a school of Levi's students would support this possibility.

The two colophons with their translations are shown in Figs. 1 and 2.

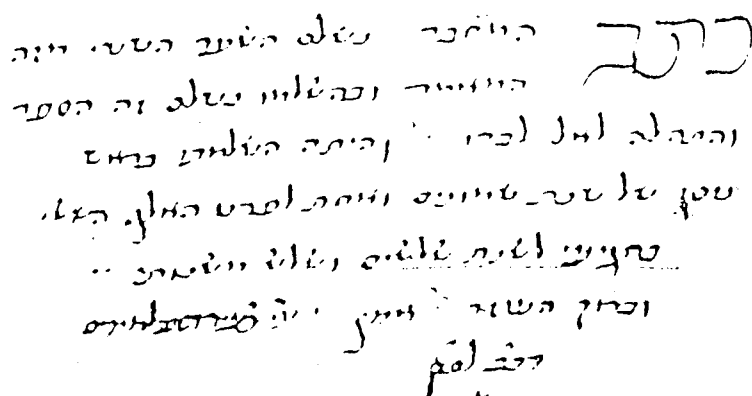


FIG. 1. Colophon from Vienna Ms. of *Maaseh Hoshev*, representing the first edition. The author writes: the sixth section of this volume is complete, and with its completion, the book is complete. The praise goes exclusively to God. Its completion was at the start of Nissan of the 81st year of the 6th millenium, when I reached the 33rd of my years. Bless the Helper.

נאם השם הזה ויחזק קולו ויבאנו נאם
 זו חסד ויחזק אל לבנו ויבאנו
 חזק אל אל שבע שנים וארבע
 לחזק קולו הזה ויחזק
 קולו

FIG. 2. Colophon from Moscow 1063 Ms. of *Maaseh Hoshev*, representing the second edition. The sixth section of this volume is complete, and with its completion, the book is complete. The praise goes exclusively to God. Its completion was in the month of Elul of the 82nd year of the 6th millenium. Bless the Helper.

Twelve Extant Manuscripts

There are 12 extant mss. of *Maaseh Hoshev*, listed below according to their current locations. Nine represent the first edition and three represent the second edition. In the first edition there are 21 problems and in the second edition there are 19. The first 13 problems, numbered 1–13, and the last four, numbered 18–21, are the same in both editions. Problems 14 and 15 in the second edition are a new presentation of the material in problems 14 and 15 of the first edition. Problems 16 and 17 of the first edition do not appear in the second edition.

The information presented here on dates, scripts, and completeness is based on a combination of my own research, the catalogs at the Institute of Microfilmed Hebrew Manuscripts in the Jewish National University Library, and personal communication with Malachi Beit-Arié, Ludwig Jesselson Professor of Codicology and Paleography at the Hebrew University of Jerusalem.

I list the mss. below in order of completeness with respect to the problem section of *Maaseh Hoshev*. The abbreviation used for reference in the critical apparatus appears in parenthesis to the right of each location. For each ms., information is given about the date, margins, similarity to other mss., and content of missing or omitted material. In general, when I say “missing” I mean that the material was included by the scribe, but the pages were later lost, while “omitted” means that the material was not included by the scribe. Within edition one, I identify two families of mss. by their consistent agreement on dozens of phrases and minor errors. A stemma summarizing the notes below appears at the end. Note that the information for the stemma is extracted mainly from the problem section of each ms. Therefore the mss. in which much of the problems are missing or omitted cannot be reliably identified until the completion of a new complete critical edition for the whole book.

Edition One: Nine Extant Mss.

- Part one: Sixty-eight Theorems and Proofs.
- Part two: Six Sections of Algorithms.
- Problems: Twenty-one Problems.
- Colophon: Nissan 5081 (Spring 1321). The Author Writes... When I Reached My 33rd Year.

Vatican (Va). Missing all of the Problem Section. Missing the end of part one, all of part two, the problem section, and the colophon. Italian script. Estimated date, late 14th century.

Munich 36 (Mu36). Missing all of the Problem Section. Omits the last third of part one, the last third of part two, and all of the problem section and the colophon. This ms. is only nine pages of very small script and bound with a larger book of mathematics including a Hebrew translation of Euclid. It seems to be someone's personal abridged version. Note that the most difficult and interesting parts of the book are missing, including proofs by induction, solutions of certain simultaneous linear equations, all the combinatorics, square and cube root extraction, and proportions. No margin notes. Sephardic script. Estimated date, 15th century.

Munich 68 (Mu68). Contains only Problems 18–20. Omits the last one and a half sections of part two and most of the problem section, then continues with the last few problems of the problem section, and concludes with the first edition colophon. The omissions here are less consistent thematically than those in Munich 36 and seem more likely to correspond to a block of missing pages in the original copied source. The ms. is similar to Vienna, Pm2462 and London. No margin notes. Ashkenazic script. Dated 1552.

New York (NY). Contains Problems 1–middle of 17. Missing the first 90% of part one, the end of the problem section, and the colophon. The last page (two sides) in the volume is misplaced and is really part of section five. A fragment of *Sefer Hamispar* by Abraham Ibn Ezra is bound in front. It was purchased by Ephraim Deinnard, a well known Hebrew manuscript dealer, in the early 1900s in Hebron. It is quite different from the other mss. and not faithful to the particular wording of the original, consistently omitting or replacing words and taking great liberty in paraphrasing. It also omits all figures, not even leaving empty space. It is similar to Pm2271 and Pa in its errors. Very few margin notes. Sephardic script with Byzantine elements. Estimated date, late 15th/early 16th century.

London (Lo). Contains Problems 1–middle of 17 (one more paragraph than NY). Missing the very end of the problem section and the colophon. It is the most reliable of the London/Vienna/Pm2462/Mu68 family. There are a large number of margin comments and corrections, many by Mordecai Finzi. Sephardic script. Estimated date, late 14th/early 15th century.

Parma 2462 (Pm2462). Contains Problems 1–middle of 20. Missing the very end of the problem section and the colophon. Believed to have been produced in Italy in the early 1400s (watermark identical to a 1426 ms). Part of the London/Pm2462/Mu68/Vi family. It is likely that Parma 2462 was an earlier source because it has fewer errors and contains some phrases that are omitted in Vienna and London. No margin notes. Semisquare Provençal script. Estimated date, late 14th/early 15th century.

Vienna (Vi). Contains all first edition problems. Complete first edition. Used as Lange's main source in the 1909 critical edition. It is the least reliable of the London/Vienna/Pm2462/Mu68 family. This ms. is not likely the source of the other mss. in the family, because it contains some hard-to-distinguish letters. If it were the source of the other mss. in the family, one would expect more errors in these letters in the other mss. than are actually found. No margin notes. Ashkenazic script. Dated 1462.

Paris (Pa). Contains all first edition problems. Complete first edition but the colophon is replaced with a standard short version with no dates or details. This ms. was brought to the library of the Oratory in 1620 and remained there until the Revolution, when it was moved to the Bibliothèque Nationale. All figures are omitted and no empty space is left.

It shares features of both the NY, Pm2271 family and the London/Pm2462/Vienna/Mu68 group but is generally much closer to the former. Very few margin notes. Sephardic script with Italian elements. Estimated date, 15th century.

Parma 2271 (Pm2271). Contains all first edition problems. Complete first edition. Believed to have been produced in Provence in the late 1300s, making this the oldest extant ms. The innermost bifolium of the last quire is bound in backward, resulting in the third and fourth to last pages being interchanged. This implies that the last eight sides of the ms. are numbered 34125678 instead of 12345678, where the eighth (and last) side is blank. Part of the Pa/NY family, but more reliable than either. Figures are in the margins with a few omitted. No margin notes. Provençal script. Estimated date, late 14th century.

Edition Two: Three Extant Mss.

Part one: Sixty-four Theorems and Proofs. Two Additions, Six Omissions, and Some Minor Variations.

Part two: Six Sections of Algorithms with Minor Additions and Omissions.

Problems: Nineteen Problems. Two Omissions, Two Major Variations, and Some Minor Variations.

Colophon: Elul 5082 (Fall 1322). No Reference to Levi's Authorship or Age.

All three second edition mss. share the features above, distinct from the first edition mss.

Moscow 30 (Mo30). Contains all second edition problems. Complete second edition. Similar to but much less reliable than Mo1063. Bound with a collection of other mathematical works copied by the same scribe, Gad Ashtrok. No margin notes. Sephardic script with Italian elements. Dated 1503.

Jerusalem (Je). Contains all second edition problems. Complete second edition. Reliable version. Very few margin notes. Space is left for all figures, but none are filled in. Sephardic script, possibly Provençal. Dated 1410.

Moscow 1063 (Mo1063). Contains all second edition problems. Complete second edition. It includes many margin notes including the insertion of much missing text from the first edition. This makes it perhaps the most reliable of all the mss. Similar to Mo30. Provençal Script. Estimated date, late 14th/early 15th century.

Stemma of the Twelve Manuscripts of Maaseh Hoshev

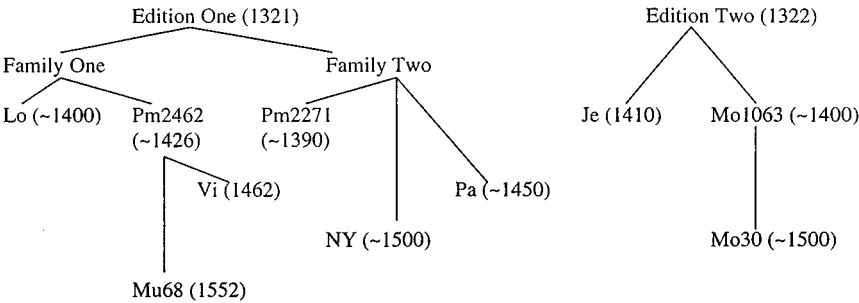
A stemma based on all the information above is shown below. The dates of the mss. are shown (some are approximate, indicated by ~) and correspond to the depth in the tree, where each level is about 50 years. None of the mss. represents an autograph, and none is completely error-free. The two editions are in separate trees. In edition one, there are two families of mss. The first family contains the Vi, Pm2462, Mu68, and Lo mss., while the second family contains the Pm2271, Pa and NY mss. The problem section is completely missing from Va and omitted in Mu36; hence for the purposes of this study they are left out of the stemma. A more complete stemma must wait until the publication of a new critical edition of the whole book, which would include the portions in Va and Mu36.

The two families of edition one are identified by dozens of similar errors, omissions, and particular choices of phrases. Within the first family, Vi, Pm2462, and Mu68 are particularly

similar throughout. Lo shares most of the features of the family but there are many places where it is correct and the other members of the family have errors. Furthermore, Lo has missing phrases which appear in the other members of the family. Hence it is likely that Lo represents a parallel branch down from the archetype of family one, edition one. On the other hand, the other three mss. in the family are virtually identical except for very minor differences, most of which are missing phrases in Vi which appear in Pm2462 and Mu68. Vi has a few pairs of letters that are extremely hard to distinguish throughout the ms., making it very unlikely that it was copied reliably, yet Pm2462 and Mu68 do not have any errors in these letters. Finally, there are no phrases missing from Pm2462 that appear in Vi or Mu68. Hence the stemma indicates that Pm2462 descends from the family one archetype, and both Vi and Mu68 descend from it. This is also consistent with the dates of these mss.

In the second family, Pm2271, Pa, and NY share dozens of similar minor errors and variations which are distinct from those of family one. The most striking example is that both Pm2271 and Pa have a small fragment of problem 14 of edition two, which appears just before problem 18. NY is missing all the pages from the middle of problem 17 on, so it is impossible to check that this striking feature is part of NY; nevertheless, NY does share the other similar features of family two. It should be noted, however, that NY is the most enigmatic of all the mss. It is not faithful to the precise wording of the original, substituting synonyms and alternate grammatical forms throughout. Finally, both Pa and Pm2271 contain phrases which are missing in the other ms., implying that neither is the likely source of the other. Hence the stemma indicates all three descending from the archetype family two.

In edition two, Mo30 is similar to Mo1063 but has many more errors and omissions. Je, in comparison, has a number of minor differences and one major difference. It includes two paragraphs in problem three which are absent in Mo1063 and Mo30. Hence the stemma indicates Mo30 descending from Mo1063 which in turn descends from the edition two autograph. Je descends in parallel from the edition two autograph. Of course this is consistent with the dates of these mss. Finally, it should be noted that both Je and Mo1063 are very reliable. In particular, Mo1063 has a large number of margin comments including many corrections and the insertion of missing sections, many from edition one. These insertions include the above-mentioned two paragraphs present in problem three of Je.



IV. AN OVERVIEW OF THE MISSING PROBLEMS

The mathematical theme underlying all of the missing problems is application of proportions. Although the mathematical foundations are based on Euclid, the problems and techniques themselves range from the trivial to the complex. The easier problems appear at

the start and the problems get progressively harder. The illustrations include:

- a. Linear and quadratic equations with one unknown.
- b. A variation of the well-known Cistern problem.
- c. Least common multiples and weighted average.
- d. Various simultaneous linear equations with two or more unknowns.
- e. Various business-related problems of purchase, sale, and profit.

The format of each problem is a general question, followed by a general method of solution, followed by an example, followed by an explanation or proof that often refers back to material earlier in the book. Sometimes the explanation is left out or shortened, and sometimes it is drawn out into a formal style proof. Sometimes there is more than one example, and sometimes variations or generalizations are discussed. The overall format, however, is consistent throughout. When variations of the problem are discussed or a digression ensues, then for the purpose of numbering the problems, I count a new problem only when it begins with the word *sh-ela* (question or problem).

V. HISTORICAL CONTEXT OF THE PROBLEMS

Levi's Sources

Levi presumably did not read Latin or Arabic, yet he certainly had access to Hebrew translations of Euclid and probably had access to translations of other sources. There are clear explicit and implicit references to Euclid throughout *Maaseh Hoshev*, a few appearing in the problem section.

The connection of medieval Hebrew scholars with the classical Greek works of Aristotle and Euclid is well known. What is not well known is to what extent Levi (and other medieval Hebrew scholars) had access to Latin and Arabic sources, and to what extent, if any, these sources influenced him. A comprehensive comparison of Levi's problems with other Latin and Arabic collections of problems is unquestionably important. I begin closer to home, with a comparison of a similar Hebrew book by Abraham ibn Ezra (1090–1167), a Spanish Jewish philosopher, poet, grammarian, and mathematician.

Outside of Euclid, *Masseh Hoshev* contains not the slimmest shred of an explicit or implicit reference to any other source. Nevertheless, it is interesting to note the following similarities to Abraham ibn Ezra's *Sefer Hamispar*. *Sefer Hamispar* (1146), is a book on arithmetic and calculation in seven sections with a short introduction explaining the decimal system and the use of 0 as a place keeper. (Note that both Ibn Ezra and Levi use the decimal system for integers but base 60 for fractions, as was common in that day.)

Section One:	Multiplication.
Section Two:	Division.
Section Three:	Sums.
Section Four:	Differences.
Section Five:	Fractions.
Section Six:	Ratios.
Section Seven:	Square roots.

In content, the only thing in *Maaseh Hoshev* which is completely absent in *Sefer Hamispar* is the material on combinatorics and series.

The sixth section on ratios is in style and content similar to Levi's section on proportions but not as rigorous and not as long. It also has a list of problems, which Ibn Ezra says are necessary to flush out the full power of his methods. Unlike Levi, who provided careful proofs, general examples, and special cases, Ibn Ezra explains his ideas through numerous specific examples, relying on quantity to substitute for rigor. Noteworthy in both books, in contrast to most Arabic works on arithmetic and algebra, is the absence of geometric proofs and arguments.

Although Levi makes no reference to Ibn Ezra's work, the similarities in organization and content are striking. This is especially true in the problems which appear in the sections on ratios and proportions. They both start with simple linear equations and continue with practical economic problems, such as people hiring other people to do a certain job and the latter doing only a partial job. There are of course some major differences too. Levi has 21 problems in two editions and Ibn Ezra has only 14. Levi's problems are more rigorously presented, and his later problems deal with more complicated simultaneous linear equations that Ibn Ezra does not approach. In general, Levi has more difficult problems in his list and has more innovative approaches to their solution. This is consistent with the rest of his book, which surpasses that of Ibn Ezra in ingenuity and creativity. A final difference is that all of Levi's problems are secular in nature, while Ibn Ezra includes a problem about inheritance that is religious in nature. This latter point is interesting, implying that Levi may have intended his work for a wider audience.

Levi probably had access to *Sefer Hamispar*, and because it was written in Hebrew, Levi would have been able to read it without difficulty. Nevertheless, despite the similarities in structure between *Sefer Hamispar* and *Masseh Hoshev*, one must be careful not to jump to conclusions, because there is no firm evidence or reference to back up such speculation.

Ibn Ezra was fluent in Arabic, and therefore had access to Arabic sources without the need of translation. In particular, many of Ibn Ezra's mathematics problems are in the context of business transactions, whose study in Arabic is called *mu'amalat*. There is a work by Ibn al-Haytham (11th century) [28] on *mu'amalat* which was certainly accessible and might have been available to Ibn Ezra. This work includes problems which require the manipulation of fractions and whose themes are seen in the early problems of Levi's list.

Hence Levi could have been influenced by the Arabic tradition indirectly, through Ibn Ezra. However, Levi's later problems, and certainly the rest of *Maaseh Hoshev*, go far beyond these basic themes, so this possible influence is unlikely to have been too significant.

There are other possible sources, both Latin and Arabic, that must be considered. These sources might have been available to Levi in translation, or indirectly through the work of other Hebrew scholars fluent in Arabic or Latin. However, any conclusions about a definitive source in this area are even more speculative than the connections with Ibn Ezra.

One possible Latin source is *De numeris datis* by Jordanus de Nemore [14]. Jordanus flourished in late 12th- and early 13th-century France, and his book is a compendium of about 100 algebra problems of linear, quadratic, and (only one) cubic equations with general solutions, proofs, and examples, divided into four sections. For example, the third problem in section one of his book is to solve the system of equations $x + y = a$, $xy = b$. The second section of the work is on problems described through proportions, exactly the kind of problems that Levi treats in his list of 21. Despite the fact that Jordanus' work is quite comprehensive, and that there are certainly similarities between his problems and Levi's, almost none of the problems in Levi's list correspond *directly* to any one of the hundred in

Jordanus. Here is an example that comes very close. Compare Jordanus' problem 12a from section two, with Levi's problem 20:

$$\text{Jordanus II-12a: } (x + a)/(y - b) = c, (y + d)/(x - e) = f.$$

$$\text{Levi 20: } (x + y)/(z - x) = a, (x + z)/(y - x) = b.$$

Levi's problem is indeed a special case of Jordanus'. This can be seen by setting $x = y$, $a = x$, $c = a$, $y = z$, $b = x$, $d = x$, and $e = x$. Levi's proof and discussion of this problem is also lengthier and more convoluted than Jordanus'. It is not likely that Levi would have taken a source with a more general problem and solution and presented only a special case of the problem with a more difficult proof. Furthermore, Jordanus presents all his problems directly (i.e., the words describe the equation) while Levi presents most of his problems hidden in the disguise of a business problem. The few problems in Levi that are presented directly (problems 16–21) cannot be matched exactly with any of the problems in Jordanus' list; see the example above.

Even if there were a translation of Jordanus' work into Hebrew, and there is no evidence for such a translation, it seems unlikely that Levi studied the work. Nevertheless, it is worth noting that both authors present their work in a rigorous way, with general statements and proofs supported by specific examples, and both make use of the idea of reducing one equation to a canonical form, after which it is solved using the solution to the canonical form. It should also be noted that Jordanus has a larger more comprehensive work, *De elementis arithmetice artis*, which became the standard source for theoretical arithmetic in the Middle Ages [4]. The work is broader than *Maaseh Hoshev* and includes much of the content from the list of theorems in part 1 of *Maaseh Hoshev*, except perhaps the combinatorial theorems, but there is nothing in it like the problem section presented here.

It is well known [7; 13] that Levi studied the scientific works of Ibn Rushd (in Hebrew translations), wrote supercommentary on his works, and taught students Aristotelian science. Hence one would expect that, if possible, he would have used similar sources for his mathematical work. There were Arabic works on mathematics translated to Hebrew, but some, such as the *Algebra* of Abu Kamil (850–930) [22], were translated after Levi died (by Mordecai Finzi, 15th century, in the case of Abu Kamil). Other later works, such as the *Algebra* of Omar Khayyam (1044–1123) [15], are unlikely to have been sources because they focus much more on various categories of quadratic and cubic equations than on linear equations. Levi's work in the problem section goes deeply into various linear and simultaneous linear equations, but hardly at all into higher order equations. Also, the later Arabic works have many geometric proofs and arguments completely absent in Levi's work. Levi's proofs are longwinded but contain innovative arguments which are strictly algebraic. I argue in [21] that one of Levi's contributions in the development of medieval mathematics is precisely this shift from a geometric view of algebra to a combinatorial view.

There is one work on arithmetic by al-Hassar (13th century), which was translated into Hebrew as *Sefer ha-Heshbon*, but it is unpublished and only extant in three manuscripts (Moscow, Oxford, and Vatican). The first two mss. indicate that the translator was Moses ibn Tibon, who translated the work in Montpellier in 1271. The third (Vatican) has an anonymous translator. If Levi had used an Arabic source, this would fit the bill. It was in the right place

at the right time, available in Hebrew, and the content focussed more on arithmetic and proportions than on higher order algebra.

Sefer ha-Heshbon, by al-Hassar, is a large work on arithmetic of integers and fractions, divided into many sections and subsections. The three extant manuscripts of the Hebrew translation differ greatly especially in the sections at the end. It deserves a separate study, but here we will present a brief overview in context of its possible connection to *Maaseh Hoshev*. In the book, al-Hassar discusses addition, subtraction, multiplication, division, and square and cube roots of both positive integers and fractions. He gives general methods followed by numerous particular examples to make the method clear. He gives no rigorous proofs in the book, which reads more like a practical manual of calculation than anything else. This is in great contrast to Levi's rigorous Euclidean style. Furthermore, although al-Hassar has literally hundreds of problems and examples, hardly any are presented in the puzzle style that Levi uses for his problems.

Nevertheless, there are still similarities between the two works. First of all, both books deal with similar subject matter. This is itself not surprising, since arithmetic is such a basic and practical topic. However, *Maaseh Hoshev* contains sections on sums and combinatorics whose rigorous presentation is absent from most early medieval works on arithmetic and algebra. Although *Sefer ha-Heshbon* by al-Hassar contains no combinatorics, it does contain a whole section on sums. In particular, it gives the formulae for sums of consecutive integers, consecutive odd integers, consecutive squares, and consecutive cubes. All these formulae were known well before al-Hassar's time, and some can be found in other Arabic works like those of al-Karaji (1000) and al-Samaw'al (1125–1180), but al-Hassar is later than these, and his work has a Hebrew translation. Moreover, although al-Hassar does not rigorously prove his formulae as Levi does, he does give constructive examples from which proofs could easily be derived.

Overall, *Maaseh Hoshev* is broader in scope, more rigorous, and more ingenious than *Sefer ha-Heshbon*. However, the latter could have been a source for the former, especially for the sections on square roots, cube roots, and sums, whose details are not found in Euclid. Unfortunately, as far as our problem section goes, there is little in al-Hassar's work that could have served as source material for Levi.

Who Studied Maaseh Hoshev?

Although Glasner [13] argues convincingly for a school of Levi, and identifies possible students, there is no direct evidence that there were students of *Maaseh Hoshev*. I argued earlier for the possibility of students being involved in the writing of the second edition, but the only direct evidence for this is the omission of Levi's name from the second edition colophon. Despite the existence of 12 extant mss., it is not likely that many people studied and understood everything in *Maaseh Hoshev*. In every ms., the kinds of errors that appear imply that the scribes themselves were hired copyists, rather than students. The errors show no understanding of the underlying content and often render a section completely unreadable. Without access to multiple copies of the work, these sections are not possible to reconstruct. Problem 17, in particular, which appears only in the first edition, is so badly mangled in every ms. that no one in the past 600 years was likely able to work through all the details.

On the other hand, there were at least a few serious readers of the text. I note that in Mo1063, a second edition ms., there are extensive comments in the margins. These

comments include a great deal of the missing material from the first edition, as well as context-sensitive corrections. This implies that the text was studied and cross-referenced with access to mss. of both editions. The Lo ms. also has extensive corrections in the margins, some by Mordecai Finzi, implying a careful study. Nevertheless, it should be noted that there are no direct known references to *Maaseh Hoshev* in the literature until 1909.

VI. INTRODUCTION TO THE TRANSLATION

I attempt to balance faithfulness to the author's language and style with readability. Despite Levi's genius, his style is tedious, repetitive, and ponderous. I do not wish to burden the reader, but neither do I wish to hide the natural flow of his style.

I have freely added punctuation for readability. Levi uses virtually no punctuation, nor does he number his problems as he does his theorems in part one. The numbering of the problems is ours and serves as an index for referencing. Commentary is added to clarify points about content, language, translation, and cultural references. To make it easier for the reader, I placed the commentary directly under the section being discussed, rather than in a separate section with cross references. The commentary is distinguishable from the translation, by the indentation and the smaller size font. The problems that are specific to one edition but not the other are marked Ed. 1 or Ed. 2 at the start of the problem.

Errors and Corrections

All errors are discussed in the commentary. It is not always easy to tell when an error is the result of the scribe or when it is original with Levi, although generally the errors are of the former type. The commentary distinguishes between these two kinds of error and attempts whenever possible to suggest plausible explanations for the origin of the error. When the mss. all agree on an error, I leave the error in the text, using the commentary to discuss alternative corrections. When each ms. presents a different incorrect version, I reconstruct the best correct version in the text itself, leaving the critical apparatus as a record of the errors.

Standard Hebrew Numbering

With regard to Hebrew numbering, some explanation is necessary. The standard Hebrew numbering scheme, still in use today for many purposes, is a mix of positional and symbolic styles using the 22 letters of the Hebrew alphabet. The first 9 letters of the alphabet indicate the numbers 1 through 9. The next 9 letters indicate the numbers 10 through 90. The next 4 indicate the numbers 100 through 400. Any number up to 499 is uniquely represented using the appropriate letters for each base-10 digit. A zero digit is indicated by having no letter at all. There are some exceptions to this, one of which is followed by Levi, and that is not to use the 10th letter followed by the 5th letter for 15, because this spells the name of God. Instead he uses the 9th letter followed by the 6th. For example, 115 is the 19th letter (value 100) followed by the 9th and the 6th; 207 is the 20th letter (value 200) followed by the 7th. When the need comes for numbers larger than 499, the largest possible valued letters are used. For example, 790 would be the 22nd letter (value 400) followed by the 21st letter (value 300) followed by the 18th letter (value 90); and 850 would be the 22nd letter (value 400) followed by the 22nd letter (value 400) followed by the 14th letter (value 50). When really big numbers are needed, and multiples of 400 do not suffice, the words for

“thousand” are combined with the numbering system to get anything needed. For example, 32 million would be written as the 12th letter (value 30) followed by the 2nd letter followed by “thousand thousand.”

Note that many numbers look like Hebrew words. The example of 15 and God has already been noted, but this is just one of many common words. What is more, single Hebrew letters act as prepositions or articles when added to the front of a word. These include and, with, on, to, in, and from. Hence it is ambiguous when these letters appear in front of numbers that happen to be words. One cannot tell, for example, whether “subtract 48” means to subtract our total “from” 8 or to subtract 48 from our total (the 4th letter being the preposition “from”). In order to help a reader resolve this ambiguity, small slashes or dots are made over the letters that are being used as numbers, and of course the context helps.

Levi's Notation for Numbers and Variables

Levi uses two numbering schemes for integers. One is the standard modern system, with the first nine letters of the Hebrew alphabet for the digits 1–9 and a circle for zero, and the other is the standard Hebrew system discussed above. It is not hard to differentiate what system is being used and he mixes them freely.

However, he also uses Hebrew letters as variables. He even uses sequences of Hebrew letters, to denote either the product or sum of the variables, as the context demands. Since so many more combinations of letters are allowed for variables, this exaggerates the ambiguity between variables and words. The difficulty of distinguishing leading letters that are articles or prepositions is also increased. In all these cases, the slashes are used, but it is not always easy to tell when a slash is there and when it is not. To distinguish between a sequence of variables that is a product from one which is a sum, he usually precedes the variables with the word *murkav* (composite) when it is a product or *misparei* (the numbers) when it is a sum. However, he is not always consistent about this. Levi also uses a sequence of two variables to denote one number, in the geometric sense of a line segment. When doing so, he precedes the two variables with the word *mispar* (number).

Once, in Problem 17, Levi runs out of variable names. The 22 letters of the Hebrew alphabet do not meet his needs, so he resorts to using five extra symbols. These *sofit* symbols are those used for the Hebrew letters that appear differently at the end of a word from the way they do in the middle of a word, resulting in 27 distinct symbols. This was no doubt the cause of confusion and corruption in the copying of Problem 17. The scribe could err in many new ways. Not only could he forget a slash, or omit a space, but he could write a specific letter either in its ending form, or in its normal form. This ambiguity confused the scribe who did not likely appreciate the serious error of substituting an end form for a normal form of a letter that happened to appear at the end of a list of variables. Nor did the scribe appreciate the similar serious error of substituting a normal form for an end form of a letter that happened to appear in the middle of a list of variables.

Levi writes out fractions in long hand, as in “2 of 9 parts of a dinar,” when describing fractions of things, but he also writes fractions using base 60 with the standard Hebrew system to write the numbers from 1 to 59. Spaces are left between the successive base 60 positional values. This introduces more possibilities for error, as the scribe must be careful in the spacing, to distinguish between 50; 9 and 59, which are identical in the standard Hebrew system except for the space in between the 15th letter and the 9th.

Translation of Levi's Notation

I translate numbers to equivalent modern digits when they are written as numbers either in the Hebrew or modern style, but use words when the numbers are written out as words. I translate variables using letters, using the n th letter in the English alphabet for the n th letter in the Hebrew alphabet. When a string of letters appears that means sum, literally “the numbers ABC ,” then I translate it as “the number $A + B + C$.” When the string of letters means product, I use “the product ABC ” literally as it is written.

VII. TRANSLATION OF PROBLEMS 1–15 FROM *MAASEH HOSHEV* WITH COMMENTARY

We set forth for you a variety of problems, so that you may understand similar ones.

1. A Problem Is Posed. We extract a given fraction or fractions from an unknown number and the result is a given number. What is the unknown number?

The method here is to take the prime denominator of all the fractions, to extract these fractions from this denominator, and to make a note of the result. Multiply this denominator by the second given number and divide the result by the noted value, to get the requested number.

Levi defines *moreh rishon*, or prime denominator, in Section Five of Part Two as the smallest integer that measures the denominators of the fractions, that is, the least common denominator. He often uses just the word *moreh*, which means simply denominator, in context, to denote the prime denominator or least common denominator. When he does this, I translate it as common denominator.

For example, 2 fifths plus 3 fourths plus one third of a number equals twenty, and we want to know the value of the whole number. The prime denominator of these fractions is 60. We extract these fractions from it, to get 89, and this is noted. Multiply the common denominator by twenty to get 12 hundred, divide this by the noted value, to get 13 whole and 43 of 89 parts of the whole, and that is what was requested. You can check this if you wish.

Levi's example in modern notation: $(2/5 + 3/4 + 1/3)x = 20$. He directs us to combine the fractions to get $89/60$ and says that $x = 20$ ($60/89$) = $13 + 43/89$. He explains below in great detail that 89 is to 60 as 20 is to x .

This is right, because the ratio of 2 fifths of 60, to 60, equals the ratio of 2 fifths of the unknown number, to the unknown number. Similarly, the ratio of 3 fourths of 60, to 60, equals the ratio of 3 fourths of the unknown number, to the unknown number; and the ratio of a third of 60, to 60, equals the ratio of a third of the unknown number, to the unknown number. When we add them together, the ratio of all the fractions taken from 60, to 60, equals the ratio of these all the fractions taken from the unknown number, to the unknown number. Hence, the ratio of 89 to 60 equals the ratio of 20 to the unknown number. Hence, the product of 60, which is the second, with twenty, which is the third, equals the product of 89, which is the first, with the unknown number. And use this as a model.

The references here to “first,” “second” and “third” come from Section Six of Part Two where Levi reminds us from Euclid that four numbers are related if the first times the fourth equals the second times the third. That is, $A/B = C/D$ if and only if $AD = BC$ (Book 7, Proposition 19 of *The Elements*).

2. Problem. Certain fractions of an unknown number exceed a second set of fractions of this unknown number by a given amount; what is the whole number?

Take the common denominator of all these fractions, extract from it the first set of given fractions and make a note of the result. Also extract from it the second set of given fractions and make a note of this second value. Subtract the second noted value from the first, and the remainder is the adjusted noted value. Multiply the common denominator by the given amount, divide the result by the adjusted noted value, and the result is the requested number.

For example, 3 sevenths plus 4 fifths of an unknown number exceed 2 thirds plus a fourth of the unknown number by twenty, and we want to know the value of this number. The common denominator of all these fractions is 420, and 3 sevenths plus 4 fifths of it is 516. And 2 thirds of 420 plus a fourth of it is 385. The first set of fractions exceeds the second set of fractions by 131. We multiply the common denominator by twenty and divide the result by 131, to get 64 whole and 17 of 131 parts of the whole. This is what was requested, and you can check this if you wish.

Levi's example in modern notation: $(3/7 + 4/5)x - (2/3 + 1/4)x = 20$. The "proof" below is tedious, methodical, and correct. Of course, he does not use any of the algebraic tools routinely expected of today's grade school students, substituting in their place an argument using proportions.

This is right, because the ratio of the sum of the first set of fractions extracted from 420, to 420, equals the ratio of the first set of fractions extracted from the unknown number, to the unknown number. By exchanging them, the ratio of the first set of fractions extracted from 420, to the first set of fractions extracted from the unknown number, equals the ratio of 420 to the unknown number. Similarly, the ratio of the second set of fractions extracted from 420, to the ratio of the second set of fractions extracted from the unknown number, equals the ratio of 420 to the unknown number. By exchanging them, the ratio of the first set of fractions extracted from 420, to the second set of fractions extracted from 420, equals the ratio of the first set of fractions extracted from the unknown number, to the second set of fractions extracted from the unknown number.

By separating, the ratio of the first set of fractions extracted from 420, to its excess over the second set of fractions extracted from 420, which is 131, equals the ratio of the first set of fractions extracted from the unknown number, to twenty, which is its excess over the second set of fractions extracted from the unknown number. By exchanging them, the ratio of the first set of fractions extracted from 420, to the first set of fractions extracted from the unknown number, equals the ratio of 131 to twenty. But we already showed that the ratio of the first set of fractions extracted from 420, to the first set of fractions extracted from the unknown number, equals the ratio of 420 to the unknown number. Hence, the ratio of 420 to the unknown number, equals the ratio of 131 to twenty. Hence, the product of 420 with twenty equals the product of 131 with the unknown number. And use this as a model.

3. Problem. The cost of a given quantity of a product is given. What is the cost of a second given quantity of this product?

Multiply the given cost by the second given quantity, divide by the first given quantity, and this is what was requested.

For example, the cost of 11 measures of wheat is 7 dinars, and you want to know the cost of 15 measures of wheat. Multiply 7 by 15 and divide by 11, to get 9 whole and 6 of 11 parts of one, which is what was requested. That is, the cost of 15 measures is 9 dinars and 6 of 11 parts of a dinar.

This is right, because the ratio of the first quantity to the second quantity equals the ratio of the known cost to the unknown cost. This is self-evident. Thus the product of the second quantity with the known cost equals the product of the first quantity with the unknown cost. And use this as a model.

Here is the method to convert a given fraction of the day into hours and fractions of an hour, or to convert a given fraction of the litra, which is 20 dinars, into dinars and peshuts and fractions of a peshut, or to convert anything that is made up of a known number of parts.

For example, we want to know how many hours and fractions of an hour there are in 83 of 109 parts of a day. We know that the number of hours in a day is 24. So the ratio of 83 to 109 equals the ratio of the unknown number of hours to 24. Multiply 24 by 83, divide by 109, and that is what was requested. That is approximately 18 hours, 51 firsts, and 12 seconds.

Levi uses base 10 for integer parts and base 60 for fractional parts of a number. This was very common in his time. Therefore, “firsts” and “seconds” should be interpreted here as minutes and seconds. Note however, that the answer is wrong. The correct answer is approximately 18; 16, 31. In other examples of conversion to base 60 throughout these problems, Levi gets the answers precisely correct, so one should not speculate that there is some flaw in his method, but rather that the numbers were corrupted and represent scribal errors. This is the first error of several similar errors found in all the extant mss.

If the example used a litra, then since we know that the number of dinars in a litra is twenty, the ratio of 83 to 109 equals the ratio of the unknown to twenty. Multiply twenty by 83 and divide by 109, to get 15 dinars and 15 of 109 parts of a dinar. You can now find the value in peshuts by multiplying 12 by 106 and dividing by 109, to get 11 peshuts and 73 of 109 parts of a peshut, which is approximately 2 thirds of one peshut. In this way, you can handle similar things.

The passage above states explicitly that there are 20 dinars in a litra and implies that there are 12 peshuts in a dinar. A litra is a measure of weight, translated commonly as a pound, and the peshut and dinar are coins, hence there are 240 peshuts to a pound.

The peshut is almost surely the denier, a paper-thin lightweight coin of mixed silver and copper, and the standard coin of Europe for 500 years between the 8th and 13th centuries. The dinar is just as surely the gros tournois, a heavy royal French silver coin introduced by king Louis IX of France in 1266, which became the standard coin of Europe from the late 13th to the 14th century [19]. The Jewish traveler Eshtori ha-Farhi mentions the use of coins in Provence in the 14th century. Farhi writes that there is a small coin that the Arabs called a pashit, twelve of which are one “white tournois” [25]. No doubt the Arab pashit is the Hebrew peshut, and this in turn is the denier, while the white tournois is the gros tournois. There were indeed 12 deniers in a gros tournois and 240 deniers in a pound [34; 35]. Note the 12 crowns around the circumference of the coin in Fig. 3.

The terms dinar and peshut are found in the Talmud, referring to generic large and small denominations respectively. Levi seems to use the anachronistic Talmudic names for real coins of his day. Note that what was called a dinar in his day was the standard muslim gold coin, worth a great deal more than Levi’s dinar. Later on, Levi will give a problem where he talks about dinars and gold dinars. Presumably, the latter is the dinar of his day, and the former is the gros tournois.

The next four paragraphs are found in all the first edition mss. In the second edition, they are present in the Jerusalem ms., copied in the margin of Moscow 1063, and omitted from Moscow 30.

Here is the method for converting a variety of fractions to one fraction. Take the prime denominator of the various denominators of the fractions, extract the fractions from it, and the result is the fraction of the common denominator.



FIG. 3. The gros tournois. Note the 12 markings around the circumference.

For example, if you wanted to convert 3 sevenths and 4 parts of 17 to a single fraction, then take these parts from the common denominator which is 119, and the result is 79 parts of 119.

Here is the method for converting the parts of a given fraction to a unit fraction. Divide the denominator of this fraction by the numerator, and the result is what was requested.

Levi uses these last two methods throughout the problem section.

For example, if you wanted to convert 3 sevenths to a unit fraction, divide seven by 3, to get 2 and a third. This is one of 2 and a third parts of that from which we wanted to extract 3 sevenths. And use this as a model.

Note “This” in the second sentence of the last paragraph refers to $3/7$. The wording in Hebrew is also ambiguous.

4. Problem. A traveler with a uniform movement, walks a fixed distance in a given amount of time. How far will he walk in a second given amount of time?

Multiply the second given amount of time by the fixed distance, divide the result by the given amount of time, and you will get what was requested. If one or both of the times contained hours, then convert the amount of time to hours, where hours are the whole unit.

For example, the traveler traveled 7 measures, 36 firsts, and 57 seconds in 13 days, and we want to know: how far would he travel in 3 days, 17 hours, 52 firsts, and 16 seconds? We convert the days to hours so that the first given time is 312 and the second time is 89 whole, 52 firsts, and 16 seconds. We multiply 7 whole, 36 firsts, and 57 seconds by 89 whole, 52 firsts and 16 seconds. We divide the result by 312 to get 2 whole, 10 firsts, and 56 seconds. This is the number of measures that he walked in the second given amount of time.

This is an error. The correct answer is approximately 2; 11, 37. Note that the number 52 and 22 look alike in Hebrew, and that if 22 is used instead, then the answer is 2; 10, 53, which is much closer to the given answer but still incorrect. The Pm2271 ms. has 22 for the first occurrence of 52, but not for the second. It is not clear what the correct numbers should be in this example.

This is right, because in a uniform movement, equal times make equal measures. Therefore, the ratio of time to time equals the ratio of journey to journey.

You can now consider the reverse problem. That is, the problem is posed given a uniform movement where a given distance is walked in a given amount of time. How much time will

it take to walk a second given distance? Multiply the second given distance by the given time, converting the days to hours to make it easier. Divide the result by the first given distance and this is what was requested.

For example, the traveler walks 7 measures, 36 firsts, and 57 seconds in 13 days. We want to know: how much time will it take him to walk 3 measures? We multiply 3 measures by 312 whole, and divide by 7 whole, 36 firsts, and 57 seconds, to get 122 hours, 54 firsts, and 7 seconds, which is the unknown time. This is explained by the very same reason as before.

This answer is correct to the nearest second. Note that 54 firsts is 54 minutes.

5. Problem. If two travelers, one faster than the other, are traveling with uniform movements, and the distance between the faster, who is behind, and the slower, is a fixed measure; how much time will it take the faster to catch the slower?

Divide the fixed measure by the excess distance traveled by the faster over the slower in an hour. The result is the number of hours and fractions of an hour that it takes the faster to catch the slower.

For example, the faster travels 2 measures and 37 seconds in an hour, the slower travels 30 firsts and 24 seconds in an hour, and the distance between the slower, who is in front, and the faster is 29 measures and 45 firsts. The excess distance traveled by the faster over the slower in an hour is one measure, 30 firsts, and 13 seconds. We divide this into the given distance to get 19 hours, 47 firsts, and 9 seconds. The reason for this is clear from the previous discussion.

The result, 19 hours, 47 minutes, and 9 seconds, is correct to the nearest 3600th but not exact. In this case, Levi usually says that the result is a close approximation, but here he does not.

6. Problem. A certain full container has various holes in it. One of the holes lets the contents of the container drain out in a given time; a second hole lets the contents drain in a second given time; and so on for each of the holes. All the holes are opened together. How much time will it take to empty the container?

This is a famous recreational mathematics problem, variations of which have appeared from ancient times until today [33].

First, calculate what drains from each hole in one hour and add the values all together. Note the ratio of this to the full container. This ratio equals the ratio of one hour to the time needed to empty the container.

For example, a barrel has various holes: the first hole empties the full barrel in 3 days; the second hole empties the full barrel in 5 days; another hole empties the full barrel in 20 hours; and another hole empties the full barrel empties in 12 hours. Therefore, the first hole empties one of 72 parts of the barrel in an hour; the second hole, one of 120 parts; the third hole, one of 20 parts; and the fourth hole, one of 12 parts. When we add them all up, the total that empties from all the holes in an hour is 56 of 360 parts of the full barrel. We divide 360 by 56, to get 6 whole and 25 firsts and 43 seconds. Therefore, the time to empty the barrel is approximately 6 hours, 25 firsts, and 43 seconds. The reason for this is clear.

Here Levi implicitly uses the method of converting a fraction to a unit fraction, as discussed in Problem 3. Once again note that “firsts” corresponds to minutes and “seconds” corresponds to real seconds.

7. Problem. The cost of a given item is a certain number of dinars. What is the cost of a given set of fractions of this item?

Take the common denominator of all the fractions, and extract the fractions from it. Then find the cost of this result, multiply by the number of the item, and divide by the common denominator, to get what was requested.

For example, the cost of a gold dinar is 25 dinars, and we want to know: what is the cost of a dinar with half of it, 2 sevenths of it, and 3 fourths of it? The common denominator of these fractions is 28. You extract these fractions from it to get 71. You multiply 71 by 25 to get a thousand and 775, and this is the cost of these extracted fractions. Multiply this number by the size of the purchase, which is one, and divide by 28, to get 63 whole and 11 of 28 parts of one, which is the cost. That is, 63 dinars and 11 of 28 parts of a dinar. You can check this if you wish.

Here we see that Levi distinguishes between the gold dinar, which is the standard Muslim dinar, and the dinar, which is the gros tournois. See the commentary on Problem 3.

This is right, because the ratio of the fractions extracted from the common denominator to the fractions extracted from the gold dinar equals the ratio of the common denominator to one, which is the size of the item. Thus the ratio of the cost of the fractions extracted from the common denominator to the cost of the fractions extracted from the gold dinar equals the ratio of the common denominator to one, because it is clear that the ratio of item to item equals the ratio of cost to cost. Hence, the product of the cost of the extracted fractions from the common denominator with one equals the product of the cost of the extracted fractions from the dinar with the common denominator. And use this as a model.

The explanation is one and the same if the size of the item being sold is arbitrary. For example, the cost of 7 measures of wheat is 25 dinars, and we want to know: what is the cost of the wheat with half of it, and 2 sevenths of it, and 3 fourths of it? We extract the fractions from the common denominator, which is 28, to get 71. The cost of these is 253 dinars and 4 sevenths of a dinar. Multiply this by the size of the item being sold, which is 7, and divide by the common denominator, which is 28, to get 63 dinars and 11 of 28 parts of a dinar. And use this as a model.

8. Problem. A merchant sells items of different costs, and a buyer wants to purchase the same measure of each item using a given quantity of his money.

As was the standard practice, Levi describes the general case by using a particular example without loss of generality. He does this throughout the book.

This problem is the general case of: Given a, b, c, d , and M , find x such that $ax + bx + cx + dx = M$.

The method is to add up the costs per measure of each product and note the result. From each item, he should take the fraction of a measure, equal to the ratio of the total money in his hand, to the noted result. This is what was requested.

For example, a merchant is selling four drugs. The cost of the first drug is 7 peshuts per litra; the cost of the second drug is 8 peshuts per litra; the cost of the third drug is 10 peshuts per litra; the cost of the fourth drug is 15 peshuts per litra. The buyer comes with 3 dinars to buy the same weight of each. With 3 dinars and 4 peshuts he can get a litra of each one, and the ratio of 3 dinars, to 3 dinars and 4 peshuts, is 9 tenths. Therefore, he should get this much from each drug; that is, 9 tenths of a litra. The total cost is 3 dinars, and you can check this if you wish.

This problem confirms the exchange rate between the peshut and the dinar, namely that there are twelve peshuts in a dinar. This information helps identify these coins. See commentary on Problem 3.

This is right, because the ratio of what he takes from each of the litras to the litra equals the cost of what he takes from each of the litras to the cost of the litra. However, the ratio of what he takes from an individual litra to the litra equals the ratio of what he takes from each of the litras to the litra. Hence, the ratio of the cost of what he takes from an individual litra to the cost of the litra equals the ratio of the cost of what he takes from each of the litras to the cost of the litra. By adding these up, we get that the ratio of what he takes from them all together to the number of litras equals the ratio of the cost of what he takes from them all together to the cost of the number of litras. And use this as a model.

We can solve the following problem similarly. If a moneychanger wants to buy variously priced coins with gold dinars, and wants to buy the same amount of each coin, how many should he buy from each?

For example, the cost of the first coin is 3 dinars per gold dinar, the cost of the second is 5 dinars per gold dinar, and the cost of the third is 7 dinars per gold dinar. The money-changer wants to buy an equal measure of each coin with a single gold coin. With a third of the gold, he can buy a dinar's worth of the first coin; with a fifth of the gold, he can buy a dinar's worth of the other coin; and with a seventh of the gold, he can buy a dinar's worth of the other coin. Hence, with a third of the gold plus a fifth and a seventh of it, he can buy a dinar's worth of each of the coins. And the ratio of the gold to these fractions equals the ratio of what he takes from each coin to the dinar. In this way, he should take a dinar and 34 of 71 parts of a dinar from each coin. And use this accordingly as a model for similar problems.

This last example is correct but a little confusing, because the cost for coins is given in dinars per gold dinar. The gold dinar acts here as the standard measure for coins, just as a litra does for weight. That is, a gold dinar's worth of the first coin costs 3 dinars. The money-changer has one gold dinar, so he can get 3 dinars worth of the first coin for his gold dinar, or a dinar's worth for one third of his gold dinar. The final answer comes from inverting $71/105$, the sum of $1/3$, $1/5$ and $1/7$.

9. Problem. A merchant sells items of different costs. What is the smallest number of measures one can buy from each one, so that the cost of what he buys from this one equals the cost of what he buys from that one?

That is, given integers a_1, a_2, \dots, a_n , find the smallest b_i 's such that $a_1 b_1 = a_2 b_2 = \dots = a_n b_n$.

The method is to write all the costs in a row one after the other. Beneath them, always in a single row, write one number under the other, and do so for each of the numbers. After this, take the smallest numbers corresponding to these ratios, and the result under each item's cost is the number of measures that he should buy from that item.

Levi's description is also ambiguous in the Hebrew, as attested to by the half dozen or so different figures in the various mss. The ambiguity is made worse by the words "one" (אחד) and "other" (אחר), which differ by a single letter that look alike, especially in certain hands. The translation and figure that appear represent a composite which I believe is the correct interpretation. His example that follows justifies my choice. Note how the pair for each ratio is in a new line by itself, just as Levi insisted "always in a single row."

For example, a merchant sells four drugs. The cost of the first drug is 2 dinars per litra; the cost of the other is 3 dinars per litra; the cost of the other is 12 dinars per litra; the cost of the other is 20 dinars per litra. We want to know: how many litras should one buy from

each one of the drugs, so that the cost of what is bought from one equals the cost of what is bought from the other ones?

$$\begin{array}{cccc}
 \underline{2} & \underline{3} & \underline{12} & \underline{20} \\
 & 3 & 2 & \\
 & & 12 & 3 \\
 & & & 20 & 12 \\
 \\
 \underline{30} & \underline{20} & \underline{5} & \underline{3}
 \end{array}$$

In one line, write down the costs per litra of the items, which are 2, 3, 12 and 20; and always in a new line underneath, write the matter in the order we discussed before. That is, write 3 under 2, and 2 under 3. Similarly, write 12 under 3, and 3 under 12. Also, write 20 under 12, and 12 under 20. Afterwards, take the smallest numbers that relate according to these ratios, that is, the ratio of 3 to 2, the ratio of 12 to 3, and the ratio of 20 to 12. These numbers, according to the explanation of Euclid, are 30, 20, 5, and 3, meaning that the ratio of 30 to 20 equals the ratio of 3 to 2, and the ratio of 20 to 5 equals the ratio of 12 to 3, and the ratio of 5 to 3 equals the ratio of 20 to 12. And so we buy 30 litras from the drug that costs 2 dinars; and this is the number associated with that cost. We buy 20 litras from the drug that costs 3 dinars. We buy 5 litras from the drug that costs 12 dinars. We buy 3 litras from the drug that costs 20 dinars. The costs of what we buy from each one are all equal. And use this as a model.

Levi is asking for the smallest numbers x , y , z , and w such that $x/y = 3/2$, $y/z = 12/3$, and $z/w = 20/12$. That is, $2x = 3y = 12z = 20w$. The least common multiple of 2, 3, 12, and 20 is 60; dividing by 2, 3, 12, and 20, respectively, gives $x = 30$, $y = 20$, $z = 5$, and $w = 3$. At the beginning of his book, Levi warns that Books 7–9 of Euclid are prerequisite reading and that he will not review what he expects the reader to know. In the last paragraph, Levi keeps his promise, saving himself a long discussion of least common multiples. He is implicitly referring to Book 7 Propositions 33–39.

It is appropriate to stick to this method if the cost of one of the drugs is in peshuts, since you convert the costs of the other drugs to peshuts. Use this as a model for similar things.

This is right, because the ratio of 30 to 20 equals the ratio of 3 to 2. Hence, the product of 30 with 2 equals the product of 20 with 3. However, the product of 30 with 2 equals the cost of 30 litras at 2 dinars per litra; and the product of 20 with 3 equals the cost of 20 litras at 3 dinars per litra. Hence, the cost of 30 litras at 2 dinars per litra equals the cost of 20 litras at 3 dinars per litra. Similarly, the cost of 20 litras at 3 dinars per litra equals the cost of 5 litras at 12 dinars per litra, and the cost of 5 litras at 12 dinars per litra equals the cost of 3 litras at 20 dinars per litra. And use this as a model.

10. Problem. A merchant sells items of different costs, and a buyer comes to purchase a single measure from all of them, so that the cost of what he buys from one item equals the cost of what he buys from each of the other items. How much of the measure should he buy from each item? What is the cost of the measure?

The problem is the general case of: Given a , b , c , find x , y , z such that $x + y + z = 1$ and $ax = by = cz$.

It is appropriate to find the smallest number of measures to take from each item so that the costs of each are equal. When you complete this task, add the numbers together and note the sum. This sum represents the number of parts into which to divide the measure. From each item, take the number of these parts equal to the number beneath that item, and this is what was requested.

In the last sentence, Levi is referring implicitly to Problem 9, so that when he says “beneath,” he means the numbers at the bottom of the previous figure.

However, you learn the cost of the measure by adding up all the costs of these smallest amounts each according to its item, which is the cost of their combination, and dividing by the noted sum. This is what was requested.

For example, a merchant is selling 3 drugs. The cost of the first is 7 peshuts per litra; the cost of the second is 10 peshuts per litra; the cost of the third is 20 peshuts per litra; and the buyer comes to purchase a litra from all of them, where the cost of what he buys from one equals the cost of what he buys from each of the others. The smallest amounts to take from each one so that the costs are equal are 20, 14, and 7. Their sum is 41, and this is the number of parts into which to divide the litra. Buy 20 parts from the drug that costs 7 peshuts per litra; buy 14 parts from the drug that costs 10 peshuts per litra; and buy 7 parts from the drug that costs 20 peshuts per litra. All together, this is 41 parts, which is one litra.

This is one of many examples, that can be used effectively to teach problem solving, by taking the focus off the mechanical algebraic manipulation. As he does throughout, Levi solves this problem without “substituting” or “solving,” as one might do with access to modern algebraic tools.

However, the cost of the litra is found by taking the costs of the amounts 20, 14 and 7, respectively, and adding them up. The total is 35 dinars. Divide 35 dinars by 41 to get 10 peshuts and 10 of 41 parts of a peshut, which is what was requested. And use this as a model.

This is right, because the cost of 20 litras at the low price equals the cost of 14 litras at the intermediate price, which equals the cost of 7 litras at the high price. The ratio of 20 of 41 parts of a litra to 20 litras equals the ratio of 14 of 41 parts of a litra to 14 litras, which equals the ratio of 7 of 41 parts of a litra to 7 litras. And accordingly, the ratio of the cost of 20 of 41 parts of a litra to the cost of 20 litras equals the ratio of the cost of 14 of 41 parts of a litra to the cost of 14 litras, which equals the ratio of the cost of 7 of 41 parts of a litra to the cost of 7 litras. By exchanging them, the ratio of 20 of 41 parts of a litra at the first price to 14 of 41 parts of a litra at the second price equals the ratio of the cost of 20 litras at the first price to the cost of 14 litras at the second price. However, the cost of 20 litras at the first price equals the cost of 14 litras at the second price. Hence, the cost of 20 of 41 parts of a litra at the first price equals the cost of 14 of 41 parts of a litra at the second price. Similarly, the cost of 14 of 41 parts of a litra at the second price equals the cost of 7 of 41 parts of a litra at the third price. And use this as a model.

11. Problem. A merchant sells two items at different prices, and a buyer wants to purchase a single measure of the two, for a given cost which is greater than the lower price and smaller than the higher price.

The question here is implicitly: What fraction of the measure should he buy from each item? The problem is equivalent to: Given $a < b < c$, $x + y = 1$, and $ax + cy = b$, find x and y .

It is appropriate to take the excess of the higher price over the lower price, and divide the measure into this many parts. Then take the deficiency of the lower price, to find the number of parts to buy from the higher priced item; and the excess of the higher price, to find the number of parts to buy from the lower priced item.

“Deficiency” and “excess” are with respect to the given cost. His solution is: let $x = (c - b)/(c - a)$ and $y = (b - a)/(c - a)$.

For example, the merchant sells two drugs. The price of the first is 17 peshuts per litra, and the price of the second is 24 peshuts per litra. A buyer wants to buy a measure of the two at a cost of 19 peshuts. The excess of the higher price over the lower price is 7, so divide the measure into 7 parts. The difference between the lower price and 19 is two, so this is the number of parts of the measure that should be bought from the higher priced item. The excess of the higher price is five, so this is the number of parts that should be bought from the lower priced item. The cost of the measure is 19 peshuts. And use this as a model.

This is right, because here we have three different numbers: 17, 19, and 24. The product of 24 with two, which is the excess of the middle over the small, plus the product of 17 with five, which is the excess of the large over the middle, counts 19, the middle, as many times as the excess of the large over the small, which is 7.

Levi is implicitly referring to Theorem 45 in part one of the book which states that given $a < b < c$, then $c(b - a) + a(c - b) = b(c - a)$. See Appendix.

Hence, the sum of the cost of two litras at the price of 24 peshuts each, plus the cost of five litras at the price of 17 peshuts each, equals 7 times 19. The number of litras here is 7, hence each one sells for 19 peshuts. Furthermore, the ratio of two sevenths of a litra to two litras equals the ratio of five sevenths of a litra to five litras. Accordingly, the ratio of the cost of two sevenths of a litra at the higher price to the cost of two litras at the higher price equals the ratio of the cost of five sevenths of a litra at the lower price to the cost of five litras at the lower price. Accordingly, the ratio of the cost of two sevenths of a litra at the higher price plus five sevenths of a litra at the lower price to the cost of two litras at the higher price plus five litras at the lower price equals the ratio of the cost of two sevenths of a litra at the higher price to the cost of two litras at the higher price.

Levi is using the well known theorem that if $a/b = c/d$, then $(a + c)/(b + d)$ is also equal to them. He discusses this theorem and similar ones in Section f of part two.

However, the ratio of the cost of two sevenths of a litra at the higher price to the cost of two litras at the higher price is a seventh of the cost. Hence, the ratio of the cost of two sevenths of a litra at the higher price plus five sevenths of a litra at the lower price to the cost of two litras at the higher price plus five litras at the lower price is a seventh of the cost. It was already made clear that the total cost is 7 times 19. Hence, the cost of this litra is a seventh of 7 times 19, which is 19 peshuts. And use this as a model.

When Levi says that the ratio is “a seventh of the cost,” he means that the former cost is $1/7$ the latter cost. That is, the ratio itself is equal to $1/7$. Also, notice that Levi uses “litra” here instead of the more general “measure” used in the statement of the problem.

12. Problem. A merchant sells a number of items with different prices. A buyer comes to purchase a single measure from them all, whose total cost exceeds the lowest price

and is exceeded by the higher price. What parts of the measure should he buy from each item?

This is a generalization of the previous problem from two variables to n . Levi knows that his solution is not unique, and he implies this by his comment in the next paragraph “so that the answer will be as small as possible.” He also implies that any solution must include a positive measure of each item. That is, no item can be left out.

It is appropriate to pair up each one of the lower prices with one of the higher prices. If the higher prices do not suffice with respect to the lower, since the number of higher prices is either greater or smaller, then match up each of the corresponding remaining prices, with the item whose price is closest to the desired cost, so that the answer will be as small as possible. After you complete this procedure as described for all the pairs, then sum up all the excesses of the larger number over the smaller number in each pair, and divide the litra into this many parts. If there was an item whose price is equal to the desired cost, so that this item has no pair, then take one part or many parts from this item, accordingly as you wish. Add this number of parts to the sum, and divide the litra into that many parts.

For example, the merchant sells seven drugs. The price of the first drug is 3 peshuts per litra; the price of the second drug is 5 peshuts; the price of the third drug is 8 peshuts; the price of the fourth is 11 peshuts; the price of the fifth is 15 peshuts; the price of the sixth is 19 peshuts; and the price of the seventh is 28 peshuts. A buyer wants to purchase a litra from all with 15 peshuts. There are two higher prices, the drug that costs 19 peshuts per litra, and the drug that costs 28 peshuts per litra, and there are four lower prices, the drug that costs 4 peshuts, the one that costs 5 peshuts, the one that costs 8 peshuts, and the one that costs 11 peshuts. We place 3 peshuts corresponding to 19 peshuts, and 5 peshuts corresponding to 28 peshuts. Since there are leftover drugs with lower prices but none with higher prices, we take the higher price that is closest to the desired cost 15, and this is 19. We pair up 19 with each of the remaining lower prices, as you can see in this figure.

$$\begin{array}{cccc} \frac{3}{19} & \frac{5}{28} & \frac{8}{19} & \frac{11}{19} \end{array}$$

We already know how to take a litra from each of these pairs, whose cost is 15 peshuts. Accordingly, take 4 of 16 parts of a litra from drug 3, and 12 of 16 parts of a litra from drug 19. Also, take 13 of 23 parts of a litra from drug 5, and 10 of 23 parts of a litra from drug 28. Furthermore, take 4 of 11 parts of a litra from drug 8, and 7 of 11 parts of a litra from drug 19. Also, take 4 of 8 parts of a litra from drug 11, and 4 parts of 8 from drug 19. The number of all these parts is 58, so we add two parts for drug 15, which has no pair. We divide the litra up into the resulting number of parts, which is 60. We take 4 parts from drug 3, 13 parts from drug 5, 4 parts from drug 8, 4 parts from drug 11, two parts from drug 15, 23 parts from drug 19, and 10 parts from drug 28. The cost of the litra is 15 peshuts, and this is what was requested.

This is right, because it is clear from what preceded that the cost of 4 litras at price 3, plus 12 litras at price 19, equals 16 times 15. The matter is similar for each pair of corresponding numbers. Furthermore, it is obvious that the cost of 2 litras at price 15 is equal to 2 times 15. By adding everything together, the cost of all these litras, each of which is a multiple of 15, equals 60 times 15.

However, the ratio of one of 60 parts of a litra to a litra equals the ratio of the price of one of 60 parts of a litra, to the price of a litra. It is clear by our previous explanation that the ratio of the price of the sum of all the parts to the price of the sum of the litras, which is 60 times 15, equals the ratio of one to 60. Thus, the price of the parts all together is 15 peshuts, which is one of 60 parts, of 60 times 15. And use this as a model.

13. Problem. One man hires another to work a given number of days, for a fixed wage. This job requires the hiring of a certain number of men per day, each of whom leads a certain number of animals, each of which carries a given number of measures and walks a given distance. The hired man deviates from some or all of these numbers. How much should his wages be?

Take the product of all the values that were stipulated, and make a note of it. Furthermore, take the product of the actual values that were accomplished; and the ratio of the number you noted, to this product, equals the ratio of the wages he promised him, to the wages he owes him.

For example, Reuven hired Shimon to work 9 days for 10 litra. The job stipulated the hiring of 13 men each day, each of whom leads seven animals, each of which carries 15 measures and walks 6 parasas. Shimon provided 8 days, 17 men, each of whom led 6 animals, each of which carried 11 measures and walked 7 parasas. The product of the stipulated numbers, 9, 13, 7, 15, and 6, equals 73 thousand and 710, which is noted. The product of the accomplished numbers, 8, 17, 6, 11, and 7, equals 62 thousand and 832. The ratio of 10 litra to what he owes him, equals the ratio of the noted value to 62 thousand and 832. If you multiply 10 litra, the first number, by the fourth number, which is 62 thousand and 832, and you divide the result by the noted value, you will get the number of litras and fractions of a litra that he owes him. This is 8 and one half litra; and a thousand and 7 hundred and 85, of 62 thousand and 832 parts of a litra, which is 5 peshuts and 59 thousand and 850, of 73 thousand and 710 parts of a peshut.

Recall that there are 20 dinars in a litra, and 12 peshuts in a dinar.

Note the error here. The last 62,832 in the previous paragraph should be 73,710. This careless error is found in all the extant mss. There is no way to know for sure whether it is due to the scribes or to Levi himself.

This is right, because the ratio of what he owes to what he stipulated equals the ratio of what he did to what he agreed to do. And the ratio of what he did to what he agreed to do is composed of the ratios of the numbers that were stipulated to the corresponding numbers that were accomplished. This composed ratio, as we already explained, equals the ratio of the product of the stipulated numbers to the product of the numbers that were accomplished. And use this as a model.

Accordingly, this explains: if a man stipulated to another man, to fill a container of given length, width and depth with a particular item, for a given amount of money; and he filled a different container with various different dimensions from what he stipulated; how much does he owe him? Here, the ratio of the product of the three stipulated dimensions to the product of the three actual dimensions equals the ratio of the stipulated cost to the amount he owes him. And judge accordingly for similar cases.

For example, the seller sold the buyer a full container of oil for 20 dinars. The length of the container was ten measures, its width was nine, and its depth was 12. He actually

filled a container whose length was eleven, width was six, and depth was 17. The product of the former dimensions is a thousand and 80, and this is noted. The product of the actual dimensions is a thousand and 122. The ratio of the noted value to a thousand and 122 equals the ratio of 20 dinars to what he owes him.

This last example is one of many where we presumably get information about the costs of goods and services in 13th–14th century Provence.

For units of length, Levi uses the generic word for measure, rather than any specific unit. This is unlike weight and money where he uses the specific units of litra, dinar, and peshut. Perhaps units of length were not as standardized in his day as was weight or money.

(*Ed. 1*) 14. *Problem.* Reuven bought an item at a given rate of certain fractions of a measure for a certain number of dinars and sold at another rate of certain fractions of the measure for a certain number of dinars. The capital is given. We want to know if he earned or lost, and how much?

By “capital,” Levi means the amount he spent.

The method is to find the price of a measure in dinars, at each rate. It will be then be clear to you whether he lost or earned with each measure, and how many dinars per measure. The total number of measures will thereby also be clear. Multiply the total number of measures, by what he earned or lost per measure, and the result is the total number of dinars that he earned or lost.

If the problem were reversed, that is, he earned or lost a certain number of dinars, and you want to know the capital or the number of measures purchased, then divide the number of dinars that he earned or lost from the whole transaction, by what he earned or lost per measure. The result is the total number of measures purchased, and their value at the buying rate, is the capital.

A special case of this “reverse” problem shows up as problem 14 in the second edition.

For example, Reuven bought 2 fifths and 3 sevenths of a measure for 7 dinars and 8 of 11 parts of a dinar. He sold 4 ninths of a measure for 8 dinars and 3 sevenths of a dinar. The capital was a hundred dinars. We want to know whether he lost or earned, and how much?

The value of a measure at the buying rate is 9 dinars and 104 of 319 parts of a dinar. The value of a measure at the selling rate is 18 dinars and 27 of 28 parts of a dinar. Accordingly, the total number of measures purchased is 10 measures, and 21 hundred and 50, of 29 hundred and 75 parts of a measure, which is 86 of 119 parts of a measure. So he earned 9 dinars and 5 thousand 701, of 8 thousand 932 parts of a dinar, per measure. Multiply this by the number of measures, to find that he earns 103 dinars and 370 thousand and 40, of a thousand and 62 thousand 908 parts of a dinar, which is 3 thousand 190, of 9 thousand 163 parts of a dinar.

Note that “a thousand and 62 thousand 908” means 1,062,098. To appreciate this, pause before and after reading the second “thousand.”

If the problem was that he earned or lost 100 dinars in this transaction, and you wanted to know the capital, then divide 100 dinars by what he earns per measure, which in our example is 9 dinars and 5 thousand 701, of 8 thousand 932 parts of a dinar. The result is the number of measures, which is 10 measures and 32 thousand and 310, of 86 thousand and 89 parts of a measure. If you multiply the number of measures by the total price per measure at

the buying rate in dinars, the result is the capital. The reason for this is completely explained by our previous discussion.

(Ed. 2) 14. *Problem.* A man bought an item at a given rate of a certain fraction or fractions of a measure or litra, for a certain number of dinars. He sold at another rate, of certain fractions of a measure, smaller than the first, for the exact same number of dinars, and thereby made a profit. What is the capital?

This problem is a special case of the “reverse” problem from Problem 14 of the first edition. The method used here is different from the method used in the first edition. The first edition works with an idea based on the calculations of dinars per measure, while the second edition uses measures per dinar. However, both editions describe similar problems and discuss three variations of their respective original problems.

For example, Reuven bought at the rate of 2 fifths and 3 sevenths of a measure, for 2 dinars. He sold at the rate of 4 ninths for 2 dinars, and made 100 dinars profit. We want to know: what is the capital?

The method is to take the common denominator of these fractions both from the measure and from the dinars, if indeed there are fractions of either. The common denominator of these fractions in our example is 315, and that is the measure. Take 2 fifths and 3 sevenths of it, to get 216.

This is an error. The result 216 is wrong; it should be 261. Note that changing the buying rate to $2/5 + 2/7$ makes 216 correct. This is more likely the source of the error than a careless digit swap, since in the Hebrew numbering, 216 (ריו) and 261 (רטא) do not look alike. The rest of the calculation is consistent with this error, and thereby incorrect. Unlike previous errors, this one propagates forward with consistency, implying that it was likely in the original, and not the fault of the scribes.

Hence, he buys 108 of these parts for a dinar. Also, take 4 ninths of it to get 140. Hence, he sells 70 of these parts for a dinar. Accordingly, he profits 38 of these parts with each dinar. We already know that he earned 100 dinars, which is 7 thousand of these parts.

Since he sells at the rate of 70 parts per dinar, 100 dinars is “worth” 7000 parts.

It is appropriate that we take a number whose ratio to 108 equals the ratio of 7 thousand to 38, and it is already clear how to do this. If you divide this result by 70, you find the capital plus the profit together. If you divide the result by 108, which is the total number of parts that he bought for a dinar, then you get the capital. The reason for this is clear from the preceding discussion.

“This result” is the number of parts that he bought. The number of parts he bought, divided by the number of parts he can sell for a dinar, equals the value he can get for the parts bought, equals the money he received, equals the total he spent plus total profit. The number of parts he bought, divided by the number of parts he can buy for a dinar, equals the value he spent on the parts bought, which is the capital.

Notice that this method is different from the one used in the “reverse problem” of Problem 14 in the first edition. The capital in the first edition equals (total measures purchased) \times (price in dinars per measure), while in the second edition, the capital equals (total parts of measure purchased)/(parts of a measure he gets for a dinar). Perhaps this change is made in order to avoid the big fractions that appear due to the method of the first edition, or perhaps because this method generalizes more easily, as seen immediately below.

If we posed the opposite problem, that is, he lost 100 dinars by buying 4 ninths of a measure at 2 dinars and selling 2 fifths plus 3 sevenths at 2 dinars, then the method is the

same, except that when you divide the result by 70, you get the capital, and if you divide it by 108, you get what remains after the loss.

This is because the buying rate is now 70 parts per dinar, and the selling rate is 108 parts per dinar.

If you ask for the quantity he buys, then divide the result by 385, which is the number of parts in the measure, and you get the quantity he bought in measures. And use this as a model.

This is an error. The number 385 should be 315. The numbers do not look alike (פספ and פס), but perhaps the 385 comes from Problem 15 in the second edition, where $385 = 5 \times 7 \times 11$ is used in another profit/loss example.

If the number of dinars were not equal, that is, he buys a certain fraction of a measure at 3 dinars and sells some fraction of it at 25 dinars, and profits or loses 100 dinars: Using the previous method, find the parts of the common denominator that you can buy for a dinar and the parts that you can sell for a dinar. Continue in the manner of the previous method, and you will get what was requested.

This generalization is the same as the “reverse problem” of Problem 14 in the first edition.

If the parts were one and the same, but the number of dinars varied—for example, he buys 4 of 7 parts of a litra, plus 5 of 11 parts of it, for 7 dinars and 8 of 11 parts of a dinar; and he sells these parts for 9 dinars and 3 of 13 parts of a dinar; and he profits 100 dinars; and the problem posed is: what is the capital, or what is the quantity he bought?—the method is to take the common denominator of these parts of the litra and divide the litra into this many parts. You will derive the total number of parts he buys with a dinar and the total number of parts he gives for a dinar. Then continue the matter in the previous fashion, to know the capital and to know the quantity he bought.

The word (בה) “of it” in the phrase “5 of 11 parts of it” is written (’בה) which means “with 5” in all three of the second edition mss.; however, the slash on the leftmost letter is an error.

And if someone asks: What is the amount, based on a fixed measure, such that 2 sevenths plus 3 fifths of it exceeds, or is exceeded by, 4 ninths of it, by the fixed measure? The method is explained in Theorem 49.

This reference reflects the second edition numbering of the theorems in part one. Theorem 49 in the second edition does indeed solve this problem, while Theorem 49 in the first edition is unrelated. Please see the list of the theorems from part one in the Appendix.

(Ed. 1) 15. Problem. Reuven bought certain fractions of a measure, for a certain number of dinars, and the capital was a certain number of dinars. He sold part of the purchase at a different rate of certain fractions of a measure for a certain number of dinars; and he sold the rest at another rate of certain fractions of a measure for a certain number of dinars. He neither earned nor lost. We want to know: what part of the purchase did he sell at the first rate, and what part did he sell at the second rate?

This problem reduces to finding x and y , given a , b , and c , such that $x + y = c$, and $ax = by$. In this formalization, a and b are the two selling rates, and c is the quantity bought, which equals the capital divided by the buying rate.

The solution is given in the paragraph below as $x = cb/(a + b)$ and $y = c - cb/(a + b)$.

If the problem is correct, then it must be the case that he earns at the first rate and loses at the second. Find the cost of a measure at each rate, and see how much he loses per measure

at the first and how much he gains per measure at the second. The ratio of the loss to the gain equals the ratio of what he sold at the higher rate to what he sold at the lower rate. This will be clear with a slight investigation of Theorem 39 in part one.

Theorem 39 in the first edition is completely unrelated. Neither Theorem 39 in the second edition nor any other theorem seems at all related. It is not clear what Levi intended by this reference.

By adding these up, the ratio of the loss to the sum of the gain and the loss equals the ratio of what he sold at the higher rate to the size of the whole purchase. Since you know three of these, you can find the fourth in the fashion presented at the start of this section.

For example, Reuven bought at the rate of 2 sevenths of a measure for 3 dinars and a fourth of a dinar, and the capital was 100 dinars. He sold part of the purchase at the rate of 4 ninths of a measure for 6 and a third dinars. He sold the other part at the rate of 5 of 11 parts of a measure for 4 dinars and a fifth. The cost of a measure at the buying rate is 11 dinars and 3 eights of a dinar. The value of a measure at the first rate is 14 dinars and a fourth of a dinar. At the second rate, the value is 9 dinars and 6 of 25 parts of a dinar. Accordingly, the quantity purchased is 8 measures and 72 of 91 parts of a measure.

This last value equals the capital, 100, divided by the buying rate, $11 \frac{3}{8}$. The last word, "measure," is clearly the correct word. However, all first edition mss. have a careless error and use "dinar" instead. The careless error could be either original or due to scribes.

The earnings per measure, according to the first rate, is 2 dinars and 7 eighths of a dinar. The loss per measure, according to the second rate, is 2 dinars and 27 of 200 parts of a dinar. The sum of the earnings and the loss is 5 dinars and 2 of 200 parts of a dinar. Multiply the loss per measure, in dinars and fractions thereof, by the total amount purchased, in measures and fractions thereof; and divide by the sum of the earnings and the loss, in dinars and fractions thereof. The result is the total amount purchased, in measures and fractions thereof, that were sold at the higher rate. Multiply 2 whole and 27 of 200 parts of one, by 8 whole and 72 of 91 parts of one. Divide the result by 5 whole and 2 of 200 parts of one, to get the total amount purchased, in measures and fractions thereof that were sold at the higher rate. The remainder of the measures purchased were sold at the lower rate.

Many variations are now discussed.

If the problem was that he earned or lost some number of dinars, then extract the total number of measures which he earned or lost from the total number purchased, and make a note of it. Split what remains of the purchase in the previous fashion, and find the amount sold at the higher rate. The leftover is what he sold at the lower rate. If he lost, then add the amount sold at the lower rate to the noted value. If he earned, then add the noted value to the amount sold at the higher rate.

If the problem was that he sold 20 measures at the higher rate, and we want to know how many measures he should sell at the lower rate in order that he neither earns nor loses, then the ratio of 20 measures to the unknown equals the ratio of 2 dinars and 27 of 200 parts of a dinar to 20 dinars and 7 eighths of a dinar. Accordingly, you can find the unknown number in the previous fashion.

The preceding discussion explains the following. If a man trades one item for another; and the value of certain fractions of the item he offers is a certain number of dinars; and the value of different fractions of the item he receives is another number of dinars; what does he deserve to receive? For each item, find the value of one measure. The ratio of the

value of a measure for the item he receives, to the value of a measure for the item he offers, equals the ratio of the total measures he receives, to the total measures he offers. Three of these numbers are known, so you can find the fourth.

Also, if you know the sum total of the measures offered and received, you can extract how many measures he offered and how many he received. Take the price per measure for each item. The ratio of the price per measure of the item he offers, to the sum of the prices per measure of what he offers and receives, equals the ratio of the measures he receives to the sum total of all the measures he offers and receives. Three of these numbers are known, so you can find the fourth.

Also consider, if a man traded one item for two items; the price of certain fractions of a measure of the item he offers, is a certain number of dinars; the price of other fractions of a measure of one of the items he receives is given; the price of other fractions of a measure of the two items he receives is given; he offers a certain number of measures; he receives a certain number of measures of the two items; and we want to know: how many measures did he receive from the first item, and how many from the second?

Take the total value of what he offered and divide it by the number of measures he receives. This will be the combined value per measure of these two items. You already know the method for deciding what part of a measure to take from the first item, and what part from the second item, such that the value of the measure will be a fixed amount.

The method referred to in the last paragraph is from Problem 11.

Accordingly, multiply the part of the measure taken from the item one, by the number of measures taken from both items. The resulting fractions are the fractions of a measure he receives from this item, and the remainder is what he receives from the other item. The reason for this is clear from the previous discussion.

For example, Reuven gives Shimon 7 measures of a certain item, that costs 2 dinars and a seventh per measure. He receives 9 measures of two different items from Shimon; the cost of the first is a dinar and a half per measure, and the cost of the second is 2 dinars and 3 quarters per measure. The value of the 9 measures he received is equal to the value of the 7 measures he gave. We want to know: how many measures did he receive from the first item and how many did he receive from the second?

We know that the value of what he received is 15 dinars.

This is because the seven measures he gave are worth $2 \frac{1}{7}$ dinars per measure, and we are told that these seven measures total are worth the same as the nine measures he received.

Accordingly, the value per measure of what he receives is a dinar and 2 thirds. Since the value per measure of the first item is a dinar and a half, and that of the second item is 2 dinars and 3 fourths, he should take 2 of 15 parts of a measure from the item whose value is 2 dinars and 3 fourths and 13 parts of a measure from the other item. And the value of a measure is a dinar and 2 thirds of a dinar. This is made clear by converting all the fractions of a dinar to parts of the same kind, that is, the third, the half and the fourth. Thus the deficit of the smaller is 2 of these parts and the excess of the greater is 13 of these parts. Consequently, it is clear that he should take 7 measures and 4 fifths of a measure from the item whose value is a dinar and a half per measure, and take the rest from the other item.

The variations that follow are generalizations of Problem 13.

Similarly, if a man stipulated to pay another man a certain wage in dinars for a certain amount of time; and for each day that he misses, the worker pays a certain number of dinars to the hirer; the worker works a certain number of days and misses a certain number; and his loss exceeds his gain; how much time did he miss?

Find the total number of dinars that the worker earns in a working day, and the total number of dinars that he loses in a missed day. The ratio of the total dinars earned to the total dinars lost, equals the ratio of the time he missed to the time he worked. This is self-evident. Now that three of these numbers are known, you can extract the fourth from them. Also, if the sum of the two times was known, you could find out how much of the time he worked and how much he missed, in the previous fashion.

(Ed. 2) 15. Problem. A man traded an item for another. The price of the item he gave is fixed for certain fractions of a measure. The price of the item he received is fixed for certain fractions of a measure. He gave a certain measure. How much should he receive?

For example, the item he gave costs 2 dinars for 3 fifths plus 2 ninths of a measure. The item he received costs 3 dinars for 3 of 11 parts of a measure plus 2 sevenths of a measure. He gave him 20 measures. We want to know: what should he receive?

The method is to take the common denominator of these parts, both from the measure and from the dinar, to the extent that there were any such parts. For each price, find the number of parts that are worth a dinar. The ratio of these parts for the item given, to these parts for the item he received, equals the ratio of the quantity given to the quantity received. The common denominator of all these parts is 3 thousand and 465. Extract 3 fifths and 2 ninths to get 2 thousand 464.

This is an error. The number 2464 should be 2849. It would be correct if the fraction $2/9$ was changed to $3/9$. The rest of the problem is consistent with this error and thereby incorrect. Because the error propagates on, it is likely that it appeared in the original and unlikely to be the fault of the scribes.

Accordingly, for the item he gave, the number of parts per dinar is a thousand and 232. Extract 2 sevenths and 3 of 11 parts of a measure, to get a thousand and 935. Accordingly, for the item he received, the number of parts per dinar is 645. The ratio of a thousand and 232, to 645, equals the ratio of what he receives, to 20 measures; and you already know the method to get the missing number. The reason for this is clear. And use this as a model.

As in Problem 14, the distinction between Problem 15 of the first edition and Problem 15 of the second edition is that the first edition uses dinars per measure, and the second edition uses measures per dinar.

Similarly, it is clear: if a man stipulates to pay another man a certain wage in dinars, for a certain time period of work, and the worker gives up after part of the time, and his loss exceeds his gain, how much was the loss?

This is a special case of Problem 13.

For example, Reuven hired Shimon to do a job lasting 7 days and a fifth of a day, for 20 dinars. Shimon gives Reuven 4 of 11 parts of a dinar for each day he misses. Shimon worked 2 days and 3 of 7 parts of a day, gave up, and his loss exceeded his gain. We want to know: how much did he lose?

We take the common denominator of all these parts to get 385, and divide the day into this many parts.

The 385 is correct here. However, in Problem 14 of the second edition, 385 appears incorrectly in place of 315. It may have been incorrectly copied from here. See commentary there.

Then find the number of these parts for which he earns a dinar and the number of these parts for which he loses a dinar. The ratio of the number for which he earns to the number for which he loses equals the ratio of 2 days and 3 sevenths to the time he missed. He earns a dinar with each 138 parts and 12 of 20 parts of a part. He loses a dinar with each thousand and 58 parts and 3 of 4 parts of a part. And such is the ratio of 2 days and 3 of 7 parts of a day to the time he missed. And use this as a model.

Note again the consistent style in the second edition of day per dinar rather than dinar per day. Note also that Levi was not averse to using parts of a part.

And if someone asked: What is the number, such that given fractions of it equal different given fractions of a given number? The method is shown in Theorem 51.

This reference reflects the second edition numbering of the theorems in part one. Theorem 51 in the second edition does indeed solve this problem. Theorem 51 in the first edition is unrelated. Please see the list of the theorems from part one in the Appendix.

VIII. A CRITICAL EDITION OF PROBLEMS 1–15

We present a critical edition in Hebrew of Problems 1–15 using all 12 extant mss. The critical apparatus appears only in the Hebrew. The English translation and commentary repeat and expand on the important points, but do not duplicate every dull detail. On the other hand, the commentary, which appears only in English, is often necessary to decipher the difficult passages. Any errors found in all extant mss. are footnoted, and commentary on these can be found in the translation.

The critical apparatus attempts to be comprehensive but for practical considerations I leave out certain types of changes that vary widely and randomly and do not affect the meaning of the passage. These include variations of feminine and masculine forms in verbs and nouns, variations of writing numbers 1 through 10 as a word or as a letter, and trivial variations in abbreviations, spelling, or plural formations (for example, *mem* versus *nun*). Despite this, the apparatus is still quite dense and includes many variations that may not interest the casual reader. If the scribe made correct corrections or changes, then these are not noted. However, if the correct corrections were made by a subsequent owner or editor, then the mistake or omission and the correction are footnoted. Various tables and figures are found in some of the mss. Because of the broad variation and the fact that these are not textual, I present the best composite.

Footnotes appear with comments at the bottom of each page. All punctuation, paragraphs, and numbering have been added for the sake of the reader and to allow easier cross-referencing with the translation. Slashes or marks above letters are reproduced carefully, with letters that represent numbers printed normally, and letters that represent variables printed in bold; see the discussion in section VI. These marks are also used at the end of a word that is abbreviated. This happens often with the words *dinar*, *litra*, and *peshut* and their plural forms.

והנה נסדר לך¹ שאלות מתחלפות עד שתבין מהם כל מה שידמה להם².

1. **שאל שואל:** לקחנו חלק מונח או חלקים מונחים ממספר נעלם, והיה מספר מונח, כמה מספר⁴ הנעלם? הדרך בזה שתקח⁵ המורה הראשון לכל החלקים, ותקח מהמורה החלקים ההם, והעולה הוא השמור. ערוך המורה⁶ על המספר המונח השני, וחלק העולה על השמור, והנה המספר⁷ המבוקש⁸.

דמיון זה, שיהיו ב' חמישיות וג' רביעיות ושלישית מספר⁹ מה, עשרים¹⁰. ורצינו לדעת¹¹ כמה¹² כל המספר. הנה¹³ המורה לכל אלו החלקים הוא ס', לקחנו ממנו אלו החלקים ועלה פ'ט¹⁴ והוא השמור. ערכנו המורה על עשרים ועלה יב' מאות. חלקנו על השמור¹⁵, ועלה יג' שלמים ומג' חלקים מפ'ט¹⁶ באחד שלם, וככה המבוקש¹⁷. ואם תרצה תוכל לבחון זה.

והיה זה כן, לפי שיחס ב' חמישיות ס' אל ס' כיחס ב' חמישיות¹⁸ המספר הנעלם¹⁹ אל המספר הנעלם, וכן יתבאר²⁰ שיחס ג' רביעיות²¹ ס' אל ס' כיחס ג' רביעיות²² המספר הנעלם אל המספר הנעלם, ושיחס שלישית ס' אל ס' כיחס שלישית המספר²³ הנעלם²⁴ אל מספר²⁵ הנעלם. וכאשר קבצנו, הנה²⁶ יהיה יחס כל החלקים האלו²⁷ חלקוהים מס' אל ס' כמו יחס כל החלקים האלו²⁸ חלקוהים ממספר²⁹ הנעלם אל המספר הנעלם. אם כן, יחס פ'ט' אל ס' הוא כיחס עשרים אל המספר הנעלם. אם כן, שטח ס' שהוא השני, בעשרים שהוא השלישי, כמו שטח פ'ט' שהוא הראשון³⁰, במספר הנעלם. והקש על זה.

2. **שאלה:** חלקים ככה מונחים ממספר³¹ נעלם מוסיפים על חלקים מונחים שניים מהמספר הנעלם, מספר מונח. כמה כל המספר? קח המורה אל כל³² החלקים, וקח ממנו³³ החלקים המונחים הראשונים ושמור. גם קח³⁴ ממנו החלקים המונחים השניים והוא השמור השני. הוצא³⁵ השמור השני מהשמור הראשון, והנשאר הוא השמור המתוקן. ערוך המורה על המספר³⁶ המונח, וחלק העולה על השמור המתוקן, והעולה הוא המבוקש³⁷.

דמיון זה, שיהיו ג' שביעיות וד' חמישיות המספר הנעלם מוסיפים על ב' שלישיות ורביעית המספר הנעלם, עשרים. ורצינו לדעת³⁸ כמה המספר. והנה המורה³⁹ לכל אלו החלקים⁴⁰ הוא תכ'. וג' שביעיות עם ד' חמישיות הוא תק'יו, וב' שלישיות תכ' ורביעיות הם שפ'ה. והנה החלקים הראשונים מוסיפים על החלקים השניים מספר קל'א. ערכנו המורה על עשרים וחלקנו העולה על קל'א ועלה ס'ד שלמים ויו' חלקים מקל'א באחד שלם, וככה המבוקש. ואם תרצה תוכל לבחון זה.

¹ Pa - Missing. ² Mu68 - This sentence is replaced with שאלות קצת נחברנו. ³ Mu68 - Omits questions 1-17. ⁴ Je, Mo30, Mo1063 - המספר. ⁵ Je, Mo30, Mo1063 - שנקח. ⁶ Pa - העולה. ⁷ Mo30 - Missing. ⁸ Pa - ממספר. ⁹ Mo1063 - והנה המספר המבוקש replaces והוא הדרוש. ¹⁰ Pa, Mo30, Mo1063, Pm2462, Lo, NY - No space between the last two words, i.e. מהעשרים. ¹¹ Mo1063 - Space is inserted in margin. ¹² NY - לידע. ¹³ Pa - Missing. ¹⁴ Vi - כמה. ¹⁵ NY - Missing. ¹⁶ Vi - כ'ט'. ¹⁷ NY - וככה המבוקש replaces והוא הדרוש. ¹⁸ NY - כ'ט'. ¹⁹ Mo30 - Missing. ²⁰ Pa - החמישיות ס' אל ס' כיחס ב' חמישיות replaces חומשי. ²¹ NY - ג' רביעיות replaces רביע. ²² NY - ג' רביעיות replaces רביע. ²³ Pa - העולה. ²⁴ Je - התבאר. ²⁵ NY - המספר. ²⁶ Pa - העולה. ²⁷ Mo30 - Missing. ²⁸ NY - מן מספר. ²⁹ NY - ראשון. ³⁰ Pm2271 - מהמספר. ³¹ Mo30 - החלקים האלו replaces החלקים. ³² NY - אל כל replaces לכל. ³³ Pm2271, NY - ממנו is inserted after כל. ³⁴ Pm2271 - הוצא. ³⁵ NY - הדרוש. ³⁶ NY - וככה החלקים replaces חלקים. ³⁷ NY - מורה. ³⁸ Pa - לידע.

והיה זה כן, לפי שיחס החלקים הראשונים מקובצים⁴¹ הלקוחים מת'כ' אל ת'כ'⁴², כיחס החלקים הראשנים⁴³ הלקוחים מהמספר הנעלם⁴⁴ אל המספר הנעלם⁴⁵. וכאשר המירונו⁴⁶, הנה יחס החלקים הראשונים הלקוחים מת'כ' אל החלקים הראשונים הלקוחים⁴⁷ מהמספר הנעלם⁴⁸, כיחס ת'כ'⁴⁹ אל המספר הנעלם. ובזה⁵⁰ התבאר שיחס החלקים השניים הלקוחים מת'כ' אל החלקים השניים הלקוחים⁵¹ מהמספר הנעלם, הוא כיחס ת'כ' אל המספר⁵² הנעלם. אם כן, יחס⁵³ החלקים הראשנים הלקוחים מת'כ' אל החלקים הראשנים הלקוחים מהמספר הנעלם, הוא כמו יחס חלקים השניים⁵⁴ הלקוחים מת'כ' אל החלקים⁵⁵ השניים הלקוחים מהמספר הנעלם.

וכאשר המירונו⁵⁶, הנה⁵⁷ יחס⁵⁸ החלקים הראשנים הלקוחים⁵⁹ מת'כ'⁶⁰ אל החלקים השניים הלקוחים מת'כ', הוא כמו יחס⁶¹ החלקים הראשנים הלקוחים מהמספר הנעלם אל החלקים השניים הלקוחים מהמספר הנעלם⁶².

וכאשר הבדלנו⁶³, הנה יחס החלקים⁶⁴ הראשנים הלקוחים מת'כ' אצל יתרונו על החלקים השניים הלקוחים מת'כ', שהוא ק'ל'א', הוא כיחס החלקים הראשונים הלקוחים מהמספר⁶⁵ הנעלם אצל עשרים⁶⁶, שהוא היתרון⁶⁷ על החלקים השניים הלקוחים מהמספר הנעלם.

וכאשר המירונו⁶⁸, הנה⁶⁹ יחס החלקים⁷⁰ הראשונים הלקוחים מת'כ' אל החלקים⁷¹ הראשונים הלקוחים מהמספר הנעלם, הוא כמו יחס ק'ל'א' אל עשרים. וכבר היה⁷² יחס החלקים⁷³ הראשונים הלקוחים מת'כ' אל החלקים⁷⁴ הראשונים הלקוחים⁷⁵ מהמספר⁷⁶ הנעלם, כיחס ת'כ' אל המספר הנעלם. אם כן, יחס ת'כ' אל המספר הנעלם⁷⁷ כיחס ק'ל'א' אל עשרים. אם כן, שטח ת'כ' בעשרים הוא כמו שטח⁷⁸ ק'ל'א' במספר הנעלם. והקש על זה.

3. שאלה: ערך⁷⁹ מספר מונח ממסחר⁸⁰ מה⁸¹, ככה⁸². כמה ערך מספר⁸³ מונח שני מהמסחר⁸⁴ ההוא? ערך הערך המונח על המספר המונח השני, וחלק על המספר המונח הראשון⁸⁵, וככה המבוקש⁸⁶.

הלקוחים מת'כ' אל ת'כ', כיחס החלקים NY - Missing⁴³. ת'ס' - Vi⁴². נקבצים - NY⁴¹.
וכהמרנו - NY⁴⁶. אל המספר הנעלם Mo30 - Missing⁴⁵. הנעלם Pa - Missing⁴⁴. הראשנים
Pm2271 - Missing⁴⁸.. מת'כ' אל החלקים הראשונים הלקוחים Mo30 - Missing⁴⁷. וכאשר המירונו
אל המספר הנעלם. וכאשר המירונו, הנה יחס החלקים הראשונים הלקוחים מת'כ' - Missing⁴⁹.
כיהס ת'כ' - Vi, Pm2462, NY - Missing⁴⁹. אל החלקים הראשונים הלקוחים מהמספר הנעלם
NY - Missing⁵⁰. מת'כ' אל החלקים השניים הלקוחים Lo - Missing⁵¹. וכזה - Je, Pm2462, NY⁵⁰.
השני - Lo⁵⁴. אם כן יחס replaces וכשהבדלנו Mo1063, Mo30 - Missing⁵³. אל המספר replaces
הראשנים הלקוחים מהמספר הנעלם, הוא כמו יחס חלקים השניים הלקוחים NY - Missing⁵⁵.
היחס - Lo⁵⁸. יהיה - NY, Lo⁵⁷. וכאשר המירונו replaces וכשהמרנו - NY⁵⁶. מת'כ' אל החלקים
מהמספר הנעלם. וכאשר המירונו, הנה יחס החלקים הראשנים Pm2271 - Missing⁵⁹.
אל Je, Vi, Pm2462 - Missing⁶². כמו יחס replaces כיהס - NY⁶¹. ת'כ' - Je - Missing⁶⁰. הלקוחים
וכאשר הבדלנו replaces וכשהבדלנו NY⁶³. החלקים השניים הלקוחים מהמספר הנעלם
Lo - Missing⁶⁴ but it is inserted in the margin. מן המספר - NY⁶⁵. Pm2271, Mo1063, NY -
NY⁷⁰. יהיה - NY⁶⁹. וכאשר המירונו replaces וכשהמרנו - NY⁶⁸. יתרון - NY, Pm2271⁶⁷. העשרים
NY - Missing⁷⁵. חלקים - NY⁷⁴. חלקים - NY⁷³. הוא - Mo30⁷². חלקים - NY⁷¹. חלקים -
כשטח - NY⁷⁸. but it is inserted in the margin. Lo - Missing⁷⁷. ממספר - NY⁷⁶. הלקוחים
Pm2271 - Missing⁷⁹. ממספר מסחר - Vi, Pm2462, Lo - Missing⁸⁰. ערך - Vi, Pm2462, Mo1063, Lo⁷⁹. כמו שטח replaces
NY - Missing⁸². ככה Pm2271, NY - Missing⁸¹. מונח - Vi, Pm2462, Lo - Missing⁸⁰. ממסחר is inserted after
הדרוש - NY⁸⁶. ראשון - Pm2271, NY - Missing⁸⁵. מהמספר - Mo1063, Mo30⁸⁴. המספר

דמיון זה, שיהיה ערך יא' ⁸⁷ מדות תבואה ז' די', ותרצה לדעת ⁸⁸ כמה ערך ט'ו' מדות תבואה. ערך ז' על ט'ו' וחלק העולה על יא', ויעלה ט' שלמים וי' ⁸⁹ חלקים מיא' באחד, וככה המבוקש ⁹⁰. רצוני ⁹¹, שערך ⁹² ט'ו' מדות הוא ט' די' וי' ⁹³ חלקים מיא' ⁹⁴ בדינר.

והיה זה כן, לפי שיחס המסחר ⁹⁵ הראשון אל המסחר ⁹⁶ השני, כיחס הערך הידוע אל הערך הנעלם, וזה מבואר בנפשו. ולזה יהיה שטח מספר המסחר השני ⁹⁷ במספר הערך הידוע, כמו שטח מספר ⁹⁸ המסחר ⁹⁹ הראשון בערך הנעלם. והקש על זה.

דרך השבת חלקים מונחים מיום אל שעות ושברי שעה, והשבת חלקים מונחים מן הליטרא, שהוא כ' ¹⁰⁰ די', אל הדינרים והפשוטים ושברי הפשו', ומה שידמה לזה מהדברים אשר הם בעלי חלקים ידועים.

והמשל, שרצינו לדעת ¹⁰¹ כמה מן השעות ¹⁰² ושברי השעה יהיו בפ'ג' ¹⁰³ חלקים מק'ט' ¹⁰⁴ ביום. והנה ידענו שמספר שעות היום ¹⁰⁵ הוא ¹⁰⁶ כ'ד'. והנה כיחס פ'ג' אל ק'ט' כן יחס מספר ¹⁰⁷ השעות הנעלם אל כ'ד'. הנה נערוך כ'ד' על ¹⁰⁸ פ'ג' ונחלק על ק'ט', והנה המבוקש ¹⁰⁹. והוא ¹¹⁰ יח' ¹¹¹ שעות ונ'א' ¹¹² ראשונים וי"ב ¹¹³ שניים ¹¹⁴ בקירוב ¹¹⁵.

ואם היה משלינו זה בליט', הנה ידענו שמספר ¹¹⁶ דינרי הליטרא הם עשרים, וכיחס פ'ג' ¹¹⁷ אל ק'ט' ¹¹⁸ כן יחס הנעלם אל עשרים. הנה נכה עשרים בפ'ג' ונחלק על ק'ט' ¹¹⁹, ויצא ¹²⁰ ט'ו' די' וק'ו' חלקים מק'ט' בדי'. וכבר תוכל לדעת ¹²¹ כמה הם מן הפשוטים, בשתערוך ¹²² י"ב על ק'ו' ותחלק על ק'ט' ¹²³. ויצא יא' פשו' ועי' ¹²⁴ חלקים מק'ט' בפשוט, שהם ב' שלישיות פשוט אחד בקירוב ¹²⁵. ובזה תנהיג מה שידמה לזה.

¹²⁶ דרך השבת חלקים מדמיוני ¹²⁷ שברים ¹²⁸ מתחלפים חלקים ממין אחד ¹²⁹: קח המורה הראשון שימנוהו כל המורים על השברים ההם המתחלפים, וממנו תקח החלקים ההם, והעולה הם חלקים מהמורה.

דמיון זה, אם ¹³⁰ רצית להשיב ¹³¹ ג' שביעיות וד' חלקים מיז' ¹³², שברים ממין אחד, הנה תקח מהמורה, שהוא ק'יט', כל אלו החלקים, והיה העולה הוא

⁸⁷ Je, Mo1063 - 'א'. Mo1063 - corrected to 'א' in margin. ⁸⁸ NY - לידע. ⁸⁹ NY - ו'. ⁹⁰ NY - הדרוש. ⁹¹ NY - רוב'. ⁹² Je, Mo30, Mo1063 - ר"ל - NY. ⁹³ Je, Mo30, Mo1063 - וככה המבוקש replaces Pm2271. ⁹⁴ Pm2271 - מט'. ⁹⁵ Mo30, M1063, Pa - המספר. ⁹⁶ Mo30, M1063, Pa - המספר. ⁹⁷ NY - Missing the next four paragraphs. ⁹⁸ Pm2271, NY - הנעלם, וזה מבואר בנפשו. ⁹⁹ Pm2271, NY - המספר. ¹⁰⁰ Pa, Pm2271, Lo - מן השעות מהשעות. ¹⁰¹ Pa, NY - לידע. ¹⁰² Je - הם. ¹⁰³ Pa, NY - פ'ג'. ¹⁰⁴ Pa, NY - מכ'. ¹⁰⁵ Pa, NY - מנין. ¹⁰⁶ Mo30, Vi, Pm2462, Lo - Missing. ¹⁰⁷ Vi, Pm2462, Lo - 'א'. ¹⁰⁸ Mo30, Pa - שהוא. ¹⁰⁹ NY - ונ'ח'. ¹¹⁰ Vi, Pm2462, Lo - 'א'. ¹¹¹ Vi, Pm2462, Lo - 'א'. ¹¹² Vi, Pm2462, Lo - 'א'. ¹¹³ NY - 'א'. ¹¹⁴ Error - This number is approximately 18; 16, 31. ¹¹⁵ Mo30, Pa - בקרוב. ¹¹⁶ Vi, Pm2462 - שמספרי. ¹¹⁷ Pa - ק'ט'. ¹¹⁸ Pa - פ'ג'. ¹¹⁹ Pa - ק'ט'ו'. ¹²⁰ NY - ויצאו. ¹²¹ NY - לידע. ¹²² NY - בקרוב. ¹²³ Je - Missing. ¹²⁴ Vi, Pm2462 - ת'ע'. ¹²⁵ Mo30, NY - ת'ע'. ¹²⁶ Mo30, Mo1063 - Missing the next four paragraphs. ¹²⁷ NY - מדמיוני שברים. ¹²⁸ Pa, Pm2462, Pm2271, Vi, Lo - ממניי. ¹²⁹ Pa - האחד. ¹³⁰ NY - Missing. ¹³¹ Pa - להשיג. ¹³² Pa - מח'.

ע'ט¹³³ חלקים מקי'ט¹³⁴.

דרך השבת החלקים¹³⁵ ממין אחד חלק אחד¹³⁶: חלק¹³⁷ המורה¹³⁸ על החלקים ההם, על מספר החלקים ההם¹³⁹, והעולה הוא המבוקש.

דמיון זה, אם רצית להשיב ג' שביעיות חלק אחד. תחלק¹⁴⁰ שבעה על ג', ויהיה העולה ב' ושליש. והוא חלק אחד מב' ושליש בדבר ההוא אשר רצית לדעת ג' שביעיותיו¹⁴¹. והקש על זה.

4. שאלה: המתנועע תנועה¹⁴² שוה, הולך¹⁴³ בזמן מונח שעור מונח מן הדרך. כמה ילך מן הדרך בזמן מונח שני? ערוך מספר¹⁴⁴ הזמן המונח השני על השעור המונח מן הדרך¹⁴⁵, והעולה חלק על מספר הזמן המונח, ויצא לך המבוקש¹⁴⁶. ואם היו שעות באחד מהזמנים¹⁴⁷ או בשניהם, השב¹⁴⁸ מספר הזמן לשעות והשעות תהיינה לאחדים שלמים.

והמשל¹⁴⁹, שיתנועע¹⁵⁰ המתנועע¹⁵¹ ב'יג'י¹⁵² ימים ז'¹⁵³ שעורים מן הדרך¹⁵⁴ ול'י' ראשונים נ'ז'¹⁵⁵ שניים. ורצינו לדעת¹⁵⁷: כמה מן הדרך¹⁵⁸ ילך בג' ימים וי'י' שעות ונ'ב' ראשונים י'י' שניים? הנה נשיב¹⁶² הימים שעות, ויהיה הזמן הראשון המונח ש'יב'¹⁶⁴, והזמן השני פ'ט' שלמים ונ'ב' ראשונים י'י' שניים. ערכנו ז' שלמים ל'י' ראשונים נ'ז' שניים על פ'ט' שלמים ונ'ב'¹⁶⁵ ראשונים י'י' שניים, ו¹⁶⁶ חלקנו העולה על ש'יב' ועלה ב'י' שלמים י' ראשונים נ'י'¹⁶⁸ שניים¹⁶⁹. והוא מספר השעורים שהלך¹⁷⁰ מן הדרך בזמן המונח השני.

והיה זה כן, לפי שהתנועה השוה¹⁷¹, תהיה בזמנים שוים שעורים שוים. ולזה הוא מבואר שיחס הזמן אל הזמן הוא יחס התנועה אל התנועה.

וכבר תפול השאלה בהפך זה. ר"ל, שישאל השואל¹⁷² במתנועע תנועה שוה, ההולך¹⁷³ שעור מונח מן הדרך¹⁷⁴ בזמן מונח. בכמה מן הזמן¹⁷⁵ ילך שעור מונח שני¹⁷⁶ מן הדרך? ואז¹⁷⁷ תערוך השעור המונח השני¹⁷⁸ מן הדרך¹⁷⁹ על הזמן המונח, וזה¹⁸⁰ בשתשיב¹⁸¹ הימים שעות, להקל מעליך. והעולה תחלק על

חלקים מ'י' שברים ממין אחד, הנה תקח מהמורה, שהוא קי'ט', כל אלו NY - Missing¹³³. מקי'ט' - NY - Pm2462, Pa, Pm2271, Lo - באחד¹³⁴. החלקים, והיה העולה הוא ע'ט'¹³⁵. Pa - Pm2462, Missing¹³⁷ חלק אחד¹³⁶. Vi - Missing¹³⁸. ה' חלקים - Pa, Pm2462, Pm2271¹³⁹. שביעיות¹⁴⁰ Je - Missing¹⁴¹ חלק - Vi, Pm2462, Lo¹⁴². על מספר החלקים ההם¹⁴³ Pa - Missing¹⁴⁴. מהמורה. המספר - Je, Mo30, Mo1063¹⁴⁵. הולך¹⁴⁶ Vi, Pa, Lo, Pm2462 - Missing¹⁴⁷. תנועה¹⁴⁸ NY - Missing¹⁴⁹. מן הזמנים¹⁵⁰ Pm2271¹⁵¹. לך המבוקש¹⁵² replaces הדרוש¹⁵³ NY¹⁵⁴. מן הדרך¹⁵⁵ replaces מהדרך¹⁵⁶ - Vi, מתנועע¹⁵⁷ NY¹⁵⁸. יתנועע¹⁵⁹ NY¹⁶⁰. המשל¹⁶¹ NY¹⁶². השב¹⁶³ על ראשונים י'י' שניים. ו'י' - Pa - Mo1063¹⁶⁴. Je - Missing¹⁶⁵. ב'יג' - Pa, Pm2462 - Missing¹⁶⁶. ו'י' - NY¹⁶⁷. ר' - Pa - Mo1063¹⁶⁸. לידע¹⁶⁹ NY¹⁷⁰. ו'י' - Vi - Missing¹⁷¹. Lo - is in the margin. המספר¹⁷² Vi, Pm2462, Lo¹⁷³. נשים¹⁷⁴ Vi - Missing¹⁷⁵. ו'י' - Pa - Missing¹⁷⁶. הנה נשיב¹⁷⁷ הימים¹⁷⁸ Pa - Missing¹⁷⁹. נ'ב' - NY¹⁸⁰. ש'יב' שלמים¹⁸¹, but ש'יב' שלמים¹⁸² שעות, ויהיה הזמן הראשון המונח ש'יב', והזמן השני פ'ט' שלמים ונ'ב' ראשונים י'י' שניים. ערכנו י'י' שלמים ל'י' ראשונים נ'ז' שניים על פ'ט' שלמים ונ'ב' ראשונים י'י' שניים, ו¹⁸³ Pa, Pm2462, Mo1063, NY, Lo¹⁸⁴. כ'י' - Pa - Mo30¹⁸⁵. Error - This number is approximately 2; 11, 37¹⁸⁶. שילך¹⁸⁷ NY¹⁸⁸. והולך¹⁸⁹ Mo30¹⁹⁰. ר"ל, שישאל השואל¹⁹¹ replaces והוא¹⁹² NY¹⁹³. אשר¹⁹⁴ Pa - Missing¹⁹⁵. ש'י' - Lo - Missing¹⁹⁶. מן הזמן¹⁹⁷ replaces מהזמן¹⁹⁸ NY¹⁹⁹. מן הדרך²⁰⁰ replaces מהדרך²⁰¹ - Pm2271²⁰². שני²⁰³ Mo30²⁰⁴. וזה²⁰⁵ Mo30²⁰⁶. ש'י' - NY²⁰⁷. ש'י' - Mo30²⁰⁸. וזה²⁰⁹ NY - Missing²¹⁰. ואז²¹¹ is in the margin. בשתשיב²¹², however, בשנשים²¹³ Mo1063, Je - Mo30²¹⁴. וזה²¹⁵ NY - Missing²¹⁶. שתשיב²¹⁷.

השעור המונח הראשון מן הדרך¹⁸², והנה המבוקש¹⁸³.

והמשל, שילך המתנועע ז'¹⁸⁴ שעורים מן הדרך¹⁸⁵ ול'ו' ראשונים נ'ז' שניים ב'יג'¹⁸⁶ ימים¹⁸⁷. ורצינו לדעת¹⁸⁸, בכמה מן הזמן¹⁸⁹ ילך ג' שעורים מן הדרך? ערכנו ג' שעורים על ש'יב' שלמים, וחלקנו העולה על ז' שלמים ל'ו' ראשונים נ'ז'¹⁹⁰ שניים, ועלה¹⁹¹ ק'כ'ב' שעות ונ'ד' ראשונים וז' שניים, והוא הזמן הנעלם. וזה מבואר מהסבה הקודמת בעינה¹⁹².

5. שאלה: אם היו שני מתנועעים תנועה שוה, והיה האחד יותר מהיר במהלכו מן האחר¹⁹³, ומרחק המהיר מהמתוני לאחריו מדה מונחת מן הדרך, בכמה מן הזמן ישיג המהיר¹⁹⁴ המתוני? חלק המדה המונחת, על יתרון מהלך המהיר על המתוני לשעה, והעולה בידך¹⁹⁵ הם השעות וחלקי השעה¹⁹⁶ אשר ישיג בהם¹⁹⁷ המהיר המתוני.

והמשל, שיהיה¹⁹⁸ מהלך המהיר¹⁹⁹ לשעה אחת, ב' שעורים ול'ו' שניים, ויהיה מהלך המתוני לשעה אחת, ל' ראשונים כ'ד' שניים. והיה מרחק המתוני מהמהיר לפניו כ'ט' שעורים ומ'ה' ראשונים. והנה יתרון מהלך המהיר על מהלך²⁰⁰ המתוני לשעה, הוא שעור אחד ול'²⁰¹ ראשונים ויג'²⁰² שניים. חלקנו עליו המרחק המונח, ועלה י'ט' שעות מ'ז' ראשונים ט'²⁰³ שניים. וזה מבואר הסבה עם מה²⁰⁴ שקדם מהדברים²⁰⁵.

6. שאלה: הכלי המלא המונח²⁰⁶ יש לו²⁰⁷ נקבים מתחלפים. יצא מאחד²⁰⁸ מהם²⁰⁹ כל מה שבכלי בזמן מונח, ומהשני יצא מה שבכלי בזמן מונח²¹⁰ שני²¹¹, וכן ימשך הענין בכל הנקבים. ונפתחו נקבי הכלי יחד. בכמה מן הזמן יורק הכלי בכללו? חקור תחלה על מה שיצא מכל אחד מהנקבים ההם בשעה אחת, וקבץ הכל. וראה יחסו אל מלא הכלי, וכפי יחסו אליו, כן יחס השעה אל הזמן שיצא בו מלא הכלי²¹².

דמיון זה, שהחבית יש בו נקבים מתחלפים. בנקב האחד²¹³ יצא מלא החבית בג' ימים; ומהנקב²¹⁴ השני²¹⁵ יצא מלא החבית בה' ימים; ובנקב²¹⁶ האחר²¹⁷ יצא מלא²¹⁸ החבית בכ' שעות; ובנקב²¹⁹ האחר²²⁰ יצא²²¹ מלא החבית ב'יב' שעות. והנה בנקב הראשון יצא מהחבית²²² בשעה אחת, חלק אחד²²³ מע'ב' חלקים במלא החבית. ובנקב²²⁴ השני²²⁵, חלק מק'כ'²²⁶; ובנקב השלישי²²⁷, חלק מכ';

¹⁸² NY - replaces מהדרך. ¹⁸³ NY - והוא הדרוש - replaces מהדרך. ¹⁸⁴ Vi, Pm2462 - replaces מהדרך. ¹⁸⁵ NY - replaces מהדרך. ¹⁸⁶ Pa - replaces מהדרך. ¹⁸⁷ Pa - replaces מהדרך. ¹⁸⁸ NY - replaces מהדרך. ¹⁸⁹ Vi, Pm2462, NY - replaces מהדרך. ¹⁹⁰ Vi, Pm2462, NY - replaces מהדרך. ¹⁹¹ Pa - replaces מהדרך. ¹⁹² Pa - replaces מהדרך. ¹⁹³ NY - replaces מהדרך. ¹⁹⁴ NY - replaces מהדרך. ¹⁹⁵ NY - replaces מהדרך. ¹⁹⁶ NY - replaces מהדרך. ¹⁹⁷ NY - replaces מהדרך. ¹⁹⁸ NY - replaces מהדרך. ¹⁹⁹ NY - replaces מהדרך. ²⁰⁰ NY - replaces מהדרך. ²⁰¹ NY - replaces מהדרך. ²⁰² NY - replaces מהדרך. ²⁰³ NY - replaces מהדרך. ²⁰⁴ NY - replaces מהדרך. ²⁰⁵ NY - replaces מהדרך. ²⁰⁶ NY - replaces מהדרך. ²⁰⁷ NY - replaces מהדרך. ²⁰⁸ NY - replaces מהדרך. ²⁰⁹ NY - replaces מהדרך. ²¹⁰ NY - replaces מהדרך. ²¹¹ NY - replaces מהדרך. ²¹² NY - replaces מהדרך. ²¹³ NY - replaces מהדרך. ²¹⁴ NY - replaces מהדרך. ²¹⁵ NY - replaces מהדרך. ²¹⁶ NY - replaces מהדרך. ²¹⁷ NY - replaces מהדרך. ²¹⁸ NY - replaces מהדרך. ²¹⁹ NY - replaces מהדרך. ²²⁰ NY - replaces מהדרך. ²²¹ NY - replaces מהדרך. ²²² NY - replaces מהדרך. ²²³ NY - replaces מהדרך. ²²⁴ NY - replaces מהדרך. ²²⁵ NY - replaces מהדרך. ²²⁶ NY - replaces מהדרך. ²²⁷ NY - replaces מהדרך.

ובנקב הרביעי²²⁸, חלק מ'ב' ²²⁹וי. וכאשר קבצנו²³⁰ זה כלו, יהיה מה שיצא מכלם יחד בשעה אחת²³¹, נ'ו²³², חלקים משיס' ²³³חלק מבלא החבית. חלקנו ש'ס' על נ'ו ועלה ו' שלמים²³⁴ כ'ה' ראשונים²³⁵ מ'ג' שניים²³⁶ והנה הזמן שיוקם בו הכלי הוא ו' שעות כ'ה' ראשונים מ'ג' שניים בקירוב²³⁷. וסבת זה מבוארת.

7. שאלה: ערך המסחר²³⁸ המונח מספר מונח²³⁹ מן הדינרין. כמה ערך חלקים מה מונחים²⁴⁰ ממנו? קח המורה אל²⁴¹ כל²⁴² החלקים, וקח ממנו החלקים ההם. והעולה, הוצא ערכו, וכפול אותו²⁴³ על מספר המסחר. וחלק העולה²⁴⁴ על המורה, וככה המבוקש²⁴⁵.

דמיון זה, שיהיה ערך דינר זהב כ'ה' די', ורצינו לדעת²⁴⁶, כמה ערך הדינר²⁴⁷ עם חציו וב' שביעיותיו וג' רביעיותיו? והנה המורה לכל אלו השברים²⁴⁸ הוא כ'ה'. לקחת ממנו אלו החלקים ועלה ע'א²⁴⁹. ערכת ע'א²⁵⁰ על כ'ה' ועלה אלף ת'שע'ה²⁵¹ שהוא²⁵² ערך החלקים הלוקחים מהמורה. ערכת זה המספר על מספר המסחר²⁵³ שהוא אחד, וחלקת העולה על כ'ה', ועלה ס'ג' שלמים²⁵⁴ ו'א' חלקים מכ'ה'²⁵⁵ באחד²⁵⁶, וככה הערך. ר"ל, ס'ג' די'²⁵⁷ ו'א' חלקים מכ'ה'²⁵⁸ בד'ל. ואם תרצה תוכל לבחון זה.

והיה זה כן, לפי שיחס החלקים הלוקחים מהמורה אל החלקים הלוקחים²⁵⁹ מדינרי זהב²⁶⁰, הוא כיחס המורה אל אחד, שהוא מספר המסחר²⁶¹. ולזה יהיה יחס ערך החלקים הלוקחים מהמורה אל ערך החלקים הלוקחים מדינרי זהב²⁶², כיחס המורה אל אחד, לפי שהוא מבואר²⁶³ שיחס המסחר אל המסחר²⁶⁴ הוא יחס הערך²⁶⁵ אל הערך. אם כן, שטח מספר ערך החלקים הלוקחים מהמורה באחד, שוה לשטח מספר ערך החלקים הלוקחים מן הדינר במורה. והקש על זה.

ואם היה²⁶⁶ המסחר²⁶⁷ מספר מה כזה²⁶⁸ בעינו יתבאר²⁶⁹. והמשל, שיהיה ערך ז' מדות תבואה כ'ה' די', ורצינו לדעת²⁷⁰: כמה ערכם עם חציים וב' שביעיותיהם וג' רביעיותיהם? לקחנו אלו החלקים²⁷¹ מהמורה שהוא כ'ה', ועלה ע'א'. והנה ערכם ר'נ'ג'²⁷² די' וד'²⁷³ שביעיות הדינר. ערכת אותם על מספר המסחר²⁷⁴ שהוא ז'²⁷⁵ וחלקת העולה על המורה שהוא כ'ה', ועלה ס'ג' די' ו'א' חלקים מכ'ה' בד'ל. והקש על זה.

²²⁸ NY - replaces וכשקבצנו. ²²⁹ Vi - מכ'. ²³⁰ Pa, Pm2271 - Missing. ²³¹ Je - An extra. ²³² Vi - נ'ז'. ²³³ Pa, Pm2271 - Missing. ²³⁴ Vi - מע'ב'. ²³⁵ Vi - מ'ג' שניים. ²³⁶ Pa, Pm2271 - Missing. ²³⁷ Mo30 - Missing. ²³⁸ Vi, Pa, Pm2271, Pm2462, Lo - המספר. ²³⁹ NY - Missing. ²⁴⁰ NY - Missing. ²⁴¹ Vi, Pm2462 - על. ²⁴² NY - replaces לכל. ²⁴³ NY - replaces אותו על מספר המסחר. ²⁴⁴ Vi, Pm2462 - Missing. ²⁴⁵ NY - replaces אותו וכפולו. ²⁴⁶ NY - ה' דינר - לידע. ²⁴⁷ Pm2271 - Missing. ²⁴⁸ NY - replaces אלו השברים שברים אלו. ²⁴⁹ Pa - Missing. ²⁵⁰ Mo30 - replaces ערכנו. ²⁵¹ Pa - Missing. ²⁵² Mo1063, Mo30, Je - והוא. ²⁵³ NY - replaces ע'א' ו'א' חלקים מכ'ה'. ²⁵⁴ Pa - חלקים. ²⁵⁵ NY - מכ'ה'. ²⁵⁶ Vi - ר"ל, ס'ג' די' ו'א' חלקים מכ'ה'. ²⁵⁷ Pa - inserted after אחד. ²⁵⁸ Pa, Pm2271 - Missing. ²⁵⁹ NY - replaces מהדינרי זהב. ²⁶⁰ Je, Pa, Pm2271 - זהב. ²⁶¹ Vi, Pm22462 - אל המסחר. ²⁶² Pa, Pm2271 - זהב. ²⁶³ NY - Missing. ²⁶⁴ Pa, Pm2271 - זהב. ²⁶⁵ NY - replaces המסחר היה. ²⁶⁶ Mo30 - is inserted after מספר. ²⁶⁷ NY - replaces החלקים חלקים אלו. ²⁶⁸ NY - לידע. ²⁶⁹ Je - התבאר. ²⁷⁰ NY - בזו. ²⁷¹ Pa, Pm2271 - מ'ג'. ²⁷² Vi, Pm2462 - מ'ג'. ²⁷³ Mo1063 - ז', but 'ד' is in the margin. ²⁷⁴ Mo30 - המספר. ²⁷⁵ Mo30 - ע'.

8. **שאלה:** הסוחר מוכר מסחרים²⁷⁶ מתחלפי²⁷⁷ הערכים כמה שיהיו, ורצה הקונה²⁷⁸ לקנות בסך מה מן הממון²⁷⁹, מדה שוה מכל²⁸⁰ אחד מהמסחרים²⁸¹. הדרך בזה שתקבץ ערכי המדה האחת לכל²⁸² המסחרים²⁸³, והעולה שמור. וכמו יחס הסך שבידו אל השמור, יקח²⁸⁴ מהמדה מכל אחד מהמסחרים²⁸⁵, והנה המבוקש²⁸⁶.

דמיון זה, שימכור²⁸⁷ הסוחר מד' סמים. ערך²⁸⁸ הסם הראשון ז' פשו²⁸⁹ הליט'; וערך הסם השני ח'²⁹⁰ פשו' הליט'; וערך הסם²⁹¹ השלישי י' פשו' הליט'; וערך הסם²⁹² הרביעי ט' פשו' הליט'²⁹³. ובא הקונה לקנות בג' די' משקל שוה מכל²⁹⁴ אחד מהסמים. והנה בג' די' וד' פשו' יקח ליט' מכל אחד. והנה יחס²⁹⁵ ג' די' אל ג' די' וד' פשו' הוא²⁹⁶ ט' עשיריות. וככה יקח מכל סם וסם²⁹⁷, ר"ל, ט' עשיריות ליטרא. ויהיה ערך²⁹⁸ הכל ג' די', ואם תרצה תוכל לבחון זה.

והיה זה כן, לפי שיחס מה שיקח מכל ליטרא מהם אל הליטרא, כיחס ערך מה שיקח מכל ליטרא מהם אל ערך הליטרא. ואולם, יחס²⁹⁹ מה שיקח מליטרא מאחד³⁰⁰ מהם אל הליטרא, הוא כיחס מה שיקח מליטרא מכל אחד³⁰¹ מהם אל הליטרא³⁰². יהיה³⁰³ אם כן, יחס ערך מה שיקח מאחד³⁰⁴ מהם אל ערך הליטרא, כיחס ערך מה שיקח מכל אחד מהם אל ערך הליטרא. וכאשר קבצנו³⁰⁵, הנה יחס מה³⁰⁶ שיקח מכלם יחד אל מספר ליטראות כמספרם, כיחס ערך מה שיקח מכלם יחד³⁰⁷ אל ערך מספר ליטראות כמספרם. והקש על זה³⁰⁸.

ומזאת השאלה³⁰⁹ יתבאר³¹⁰, אם רצה השולחני לקנות בד' זהב מטבעות מתחלפות הערכים, ורצה לקחת מכל מטבע מהם, מספר שוה. כמה יקח מכל אחד ואחד?

והמשל, שיהיה ערך³¹¹ המטבע האחד ג' די' הזהוב³¹², וערך השני ה' די' הזהוב³¹³, וערך השלישי ז' די' הזהוב³¹⁴. ורצה השולחני לקחת בזהוב אחד, שעור שוה מכל אחד מהמטבעות. והנה בשליש³¹⁶ הזהוב³¹⁷ יקנה די'³¹⁸.

מהממון - Vi²⁷⁹ איש - NY²⁷⁸. מתחלפים - Vi, Pm2462, Lo - מתחלקי - Je²⁷⁷. המסחרים - Je²⁷⁶.
מן המסחרים - Pm2462²⁸¹. מן כל - Vi²⁸⁰. מה מן הממון replaces ממון - NY. מן הממון replaces
ראה - Vi²⁸³ Pm2462. מכל - Je²⁸², Lo. מכל אחד מהמסחרים - Pa - Missing. מהמספרים - Je, Mo30 -
הדרך בזה שתקבץ ערכי המדה replaces כמה עולה אם יקח מדה מכל אחד מהמסחרים
but, הדרך בזה שתקבץ ערכי המדה האחת לכל המסחרים - Lo - Missing. האחת לכל המסחרים
it is in the margin with מכל replacing מכל. Je²⁸⁴, Vi, Pm2271, NY. קח - Pm2271²⁸⁵.
Lo, Vi, Pm2462 -
המשל ימכור - NY²⁸⁷. והנה המבוקש replaces והוא הדרוש - NY²⁸⁶. מן המסחרים - Je, Mo30 -
replaces פשו' פ' - NY²⁸⁹. וערך - Je, Mo30 - Mo1063, Je, Mo30²⁸⁸. דמיון זה, שימכור
Je²⁹⁵ - Vi, Pm2462 - בליט' - Je²⁹³. הסם Missing - NY²⁹². הסם Missing - NY²⁹¹. ח' - Missing
ויס - Missing - NY²⁹⁷. והנה יחס is inserted after יחס - Je²⁹⁶. והנה יחס replaces ויחס - NY. יחס -
Je - Pm2271, NY³⁰⁰. יחס is inserted after זה - Pa²⁹⁹. ויהיה ערך replaces והיה - Pa²⁹⁸.
הוא כיחס מה שיקח - Missing - Mo30³⁰². מליטרא מכל אחד replaces מכל ליטרא מאחד
NY - יהיה קבצנו - Missing - NY³⁰³. הליטרי - Pm2271. מליטרא מכל מהם אל הליטרא
Mo30 - Pa - Missing³⁰⁶. וכאשר קבצנו וכשקבצנו - NY³⁰⁵. מליטרא מכל אחד
ומשאלה זו - NY³⁰⁹. והקש על זה Missing - NY³⁰⁸. יחס - Pa. מכלם יחד replaces מכל אחד
ערך. is inserted after השלישי - Pa³¹¹. התבאר - Je, Mo30, Mo1063³¹⁰. ומזאת השאלה replaces
Mo30 - Pa³¹⁶. בזהוב - Mo30³¹⁵. שלישי - NY³¹⁴. בזהוב - Mo30³¹³. בזהוב - Mo30³¹².
Mo30 - Missing³¹⁸. זהוב

מהמטבע³¹⁹ האחד³²⁰, ובחמישית³²¹ הזהוב³²² יקנה די' מהמטבע האחר³²³,
ובשביעית הזהוב³²⁴ יקנה די' מהמטבע³²⁵ האחר. אם כן³²⁶, בשלישית הזהוב
וחמישיתו ושביעיתו יקח די' מכל אחד מהם³²⁷. וכמו יחס הזהוב אל אלו
החלקים³²⁸, כן יחס מה שיקח מכל אחד מהם אל הדינר. ובה³²⁹, יקח מכל
אחד³³⁰ מהם, דינר ול' חלקים מע'א' בדינר. וכפי זה תקיש³³¹ במה שידמה³³²
לזה.

9. שאלה: הסוחר מוכר מסחרים מתחלפי³³³ הערכים. כמה המספר המעט מן
המדות שילקח³³⁴ מאחד אחד³³⁵ מהם, ויהיה ערך מה שילקח מזה³³⁶ כערך³³⁷
מה שילקח מזה? הדרך בזה שתכתב ערכי המסחרים³³⁸ זה אחר זה בטור אחד.
וכנגדם, תמיד בטור אחד, המספר האחר³³⁹ תחת האחר, וכן תעשה לכל
המספרים. ואחר זה תקח קטני המספרים על יחס אלו, והעולה בידך תחת כל
אחד מן הערכים³⁴⁰ הוא מספר המדות שילקח מהמסחר³⁴¹ ההוא³⁴².

דמיון זה, שימכור³⁴³ הסוחר מארבעה³⁴⁴ סמים. ערך הסם³⁴⁵ האחד ב' די'³⁴⁶
הליטרא; וערך הסם האחר³⁴⁷ ג' די' הליט³⁴⁸, וערך הסם האחר³⁴⁹ יב' די'
הליטרא; וערך הסם³⁵⁰ האחר³⁵¹ כ' די' הליטרא³⁵². ורצינו לדעת³⁵³: כמה מן
הליטראות ילקח³⁵⁴ מאחד אחד מהסמים³⁵⁵, ויהיה ערך מה שילקח מזה כמו
ערך מה שילקח מזה?

ב' ג' יב' כ'

ג' ב' יב' כ'
יב' ג' כ' יב'

ל' כ' ה' ג'

הנה תכתב בטור אחד ערכי הליטראות³⁵⁶ והוא ב', ג', יב', כ', ובטור אחר³⁵⁷
תחתיו תמיד הענין על זה הסדור³⁵⁸ הנזכר³⁵⁹. רצוני³⁶⁰, שתכתוב תחת³⁶¹ ב' ג',
ותחת ג' ב'. וגם כן, הנה³⁶² תכתוב על זה הדרך³⁶³ תחת ג', יב', ותחת יב', ג'.

³¹⁹ Pm2271, Lo - ממטבע. ³²⁰ Pa - Missing. האחד NY. ³²¹ NY - ובחומש. ³²² Mo30 זהוב. ³²³ Pa, Pm2271, Lo -
Missing. ³²⁴ Lo - The missing phrase is in the margin. ובחמישית הזהוב יקנה די' מהמטבע האחר. ³²⁵ NY -
Mo30 זהוב. ³²⁶ Je - Missing. ³²⁷ Je - וכך. ³²⁸ NY - מהם. ³²⁹ Je, Vi - וכזה. ³³⁰ NY - מכל אחד replaces מ'א'. ³³¹ Je, Mo30,
Mo1063. הקש. ³³² NY - replaces שידמה והקש בדומה. ³³³ NY - החלקים replaces אלו החלקים. ³³⁴ Vi, Pm2462 -
שלך. ³³⁵ NY - Missing. ³³⁶ Vi, Pm2462 - אחד. ³³⁷ NY - בערך. ³³⁸ Vi, Pa, Pm2271, NY, Pm2462, Lo -
המספרים. ³³⁹ Pa, NY - אחד. ³⁴⁰ Pa - replaces מהערכים. ³⁴¹ NY - Missing. ³⁴² NY - replaces מהמסחרים ההם.
³⁴³ Vi - Missing. ³⁴⁴ NY - Missing. ³⁴⁵ NY - ד'. ³⁴⁶ NY - ימכור. ³⁴⁷ NY - מהמסחר ההוא. ³⁴⁸ NY -
הליט. ³⁴⁹ NY - replaces והאחר. ³⁵⁰ Vi, Pm2462, Lo - Missing. ³⁵¹ NY - replaces והסם האחר. ³⁵² Lo -
Missing. ³⁵³ NY - replaces והסם האחר. ³⁵⁴ Pa - יקח. ³⁵⁵ Mo30 - הסמים. ³⁵⁶ Mo30, Mo1063, Pm2271, NY, Pm2462, Lo -
החבור. ³⁵⁷ Vi, Pa, Pm2271, NY, Pm2462, Lo - החבור. ³⁵⁸ NY - על זה הסדור הנזכר. ³⁵⁹ NY - החבור. ³⁶⁰ NY -
רצוני. ³⁶¹ NY - תחת. ³⁶² NY - וגם כן, הנה תכתוב על זה הדרך. ³⁶³ Vi, Pm2462, Lo - תחת. ³⁶⁴ NY -
הנה תכתוב על זה הדרך replaces כתוב בדרך זו.

וגם כן, הנה תכתוב³⁶⁴ תחת יב', כ', ותחת כ', יב'. ואחר³⁶⁵ כן, תקח³⁶⁷ קטני המספרים המתיחסים בכמו זה היחס. ר"ל³⁶⁸ מיחס³⁶⁹ ג' אל ב', ומיחס יב' אל ג', ומיחס כ' אל יב'. והם³⁷² לפי מה שהתבאר מאקלידס³⁷³ מספר ל', כ', ה', ג'. וזה שיחס ל' אל כ' כיחס ג' אל ב', ויחס כ' אל ה' כיחס יב' אל ג', ויחס ה' אל ג' כיחס כ' אל יב'. ולזה תקח מן הסם אשר ערכו³⁷⁴ ב' די, ל' ליט', והוא המספר הנכחי לו³⁷⁵. ומהסם אשר ערכו³⁷⁶ ג' די הליט', תקח כ' ליטראות. ומהסם אשר ערכו³⁷⁷ יב' די³⁷⁸, תקח ה'³⁷⁹ ליטראות. ומהסם אשר ערכו³⁸⁰ כ' די³⁸¹ תקח ג' ליטראות. ויהיה³⁸² ערך מה שתקח מכל אחד מהם שוה. והקש על זה.

וראו שתשמר בזה הדרך³⁸³ הנזכר אם היה ערך אחד³⁸⁴ הסמים³⁸⁵ מספר הפשוטים³⁸⁶, שתשיב ערך הסמים הנשארים לפשוטים. ומזה תקיש במה שידמה לזה.

והיה זה כן, לפי שיחס ל' אל כ', הוא כיחס ג' אל ב'. אם כן, שטח ל' בב'³⁸⁷, הוא כמו שטח כ' בג'. אבל שטח ל' בב'³⁸⁸ שוה לערך ל' ליטראות מב' די הליט', ושטח כ' בג' שוה³⁸⁹ לערך כ'³⁹⁰ ליטראות מג' די הליטרא. אם כן, ערך ל' ליטראות מב' די הליטרא³⁹² שוה לערך כ' ליטרא מג' די הליטרא. ובה³⁹³ התבאר, שערך כ' ליטראות מג' די הליטרא שוה לערך ה' ליטראות מיב' דינרין³⁹⁴ הליטרא, ושערך ה'³⁹⁵ ליטראות מיב' די הליטרא³⁹⁶ הוא³⁹⁷ כמו ערך ג' ליטראות מכ' די הליטרא³⁹⁹. והקש על זה⁴⁰⁰.

10. שאלה: הסוחר מוכר מסחרים מתחלפי⁴⁰¹ הערכים, ובא הקונה לקנות מדה אחת מכלם, ויהיה ערך מה שיקח⁴⁰² מזה⁴⁰³ שוה לערך מה שיקח⁴⁰⁴ מזה. כמה מן המדה יקח מכלם? וכמה ערך המדה ההיא?

ראוי שתקח קטני המספרים שילקח מכל אחד מהם מן המדות⁴⁰⁵ ויהיה הערך שוה. וכאשר ישלם⁴⁰⁶ לך זה, קבץ מספריהם והוא השמור. והשמור הם חלקי המדה. לקח⁴⁰⁷ מכל אחד מהם מאלו החלקים כמו המספר אשר תחתיו והנה⁴⁰⁸ המבוקש⁴⁰⁹. ואולם, ערך המדה ההיא תדעהו⁴¹⁰ כשתקבץ⁴¹¹ ערכי קטני המספרים ההם⁴¹² כל אחד מגילו, והוא ערך המקובץ⁴¹³; חלקהו⁴¹⁴ על השמור, והנה⁴¹⁵ המבוקש⁴¹⁶.

הנה replaces כתוב - NY. תחת ג', יב', ותחת יב', ג'. וגם כן, הנה תכתוב³⁶⁴ Vi - Missing. על - NY³⁶⁹. רצוני - Mo30³⁶⁸. קח - NY³⁶⁷. כן. NY - Missing³⁶⁶. ואחרי - Vi, Pm2462³⁶⁵. תכתוב - NY³⁷¹. ומיחס יב' replaces ויב' - NY³⁷⁰. המתיחסים בכמו זה היחס. ר"ל, מיחס replaces יחס מה שהתבאר replaces לפי המבואר באקלידס - NY³⁷³. והנה - NY³⁷². ומיחס כ' replaces וכ' אשר replaces שערכו - NY³⁷⁶. לו. Pa - Missing³⁷⁵. אשר ערכו replaces שערכו - NY³⁷⁴. מאקלידס י. - Pa³⁷⁹. די הליטרא - Missing³⁷⁸ Je, NY, Lo - Missing³⁷⁶ Vi, Pm2271, Pm2462, NY, Lo - Missing³⁷⁷ NY. ערכו. תקח ה' ליטראות. ומהסם אשר ערכו כ' - Missing³⁸¹ Mo1063. אשר ערכו replaces שערכו - NY³⁸⁰. replaces בדרך - NY³⁸³. והיה - Lo. והנה - Vi, Pm2462³⁸². די. NY - Missing³⁸². but it is in the margin. די, NY - Missing³⁸⁷. מהפשוטים - Vi, Pa, Lo, Pm2462³⁸⁶. מהסמים - Vi³⁸⁵. אחד - Pa - Missing³⁸⁴. בזה הדרך³⁹³ מב' די הליטרא - Missing³⁹² Je, NY, Lo - Missing³⁹¹ Je - ב'. ב'. Je³⁹⁰. שוה. Pa - Missing³⁸⁹. בכ' - NY³⁸⁸. בכ'. ושערך ה' - Pa - Missing³⁹⁶. א'. - Mo30³⁹⁵. דינרין - Lo - Missing³⁹⁴. וכזה - Vi, Pa, Pm2271, Lo - Missing³⁹³ NY. כמו ערך replaces לערך - NY³⁹⁸. שוה. - Vi, Pm2462, NY, Lo - Missing³⁹⁷. ליטראות מיב' די הליטרא. Vi, Pm2462⁴⁰². מתחלפים - NY⁴⁰¹. והקש על זה - Missing⁴⁰⁰ Vi, Pm2462. הליטרא - Missing³⁹⁹ NY. מן המדות replaces מהמדות - Pa, NY⁴⁰⁵. שילקח - Vi, Pm2462⁴⁰⁴. מזה. Pa - Missing⁴⁰³. שילקח. והוא הדרוש - NY⁴⁰⁹. והוא - Mo30⁴⁰⁸. קח - NY⁴⁰⁷. וכאשר ישלם replaces וכשישלם - NY⁴⁰⁶. מהם - Pm2271⁴¹². בשתקבץ - NY, Pm2271⁴¹¹. תדע - Pa - Missing⁴¹⁰. והנה המבוקש replaces והנה replaces והוא הדרוש - NY⁴¹⁶. והוא - Mo30, Pa - Missing⁴¹⁵. חלקהו - Vi⁴¹⁴. הנקבץ - NY⁴¹³. המבוקש.

הוא כמו יחס ערך כ' ליטר' Pm2271 - Missing ⁴⁷¹ NY - מערך ⁴⁷⁰ NY. חלקים ממא' ⁴⁶⁹ Pa - Missing מהערך הראשון אל ערך י'ד' ליטר' מהערך השני. אבל ערך כ' ליטראות מהערך הראשון הוא כמו ערך י'ד' ליטר' מהערך השני. אם כן, ערך כ' חלקים ממא' בליטרא מהערך ⁴⁷³ NY. ובזה ⁴⁷² Mol063, Je - Missing. הראשון, הוא כמו ערך י'ד' חלקים ממא' בליטרא מהערך השני. ⁴⁷⁷ Lo - Missing מערך ⁴⁷⁶ NY. בליטרא ⁴⁷⁵ NY - Missing ממא' ⁴⁷⁴ Vi - Missing. כמו ערך ⁴⁷⁸ Pa - Missing מתחלפים ⁴⁷⁹ NY. וחסר ⁴⁸⁰ NY. ⁴⁸¹ Pa - Missing מספר ⁴⁸² NY. is inserted after the margin: וחסרנו המחסיר והוא יהיה - ⁴⁸³ Pm2271 קח. ⁴⁸² NY. חלפי יתרון המוסיף: ⁴⁸⁴ Pa - Missing but inserted in the margin: ⁴⁸⁵ NY. וחסר ⁴⁸⁶ NY. ⁴⁸⁷ NY - Missing יהיה המחסיר, ⁴⁸⁸ Pa - Missing על המחסיר, והם חלקי המדה. ואחר תקח מספר חסרנו המחסיר ⁴⁸⁹ Vi, Pm2462, Lo - 'ח. ⁴⁹⁰ Vi, Pm2462, Lo - 'ח. הסם ⁴⁹¹ NY - Missing מהחסר ⁴⁹² Mo30. מהמספר ⁴⁹³ Pa - Missing וערך השני ⁴⁹⁴ Mo30. האחד ⁴⁹⁵ Pa - Missing replaces השני ⁴⁹⁶ Pa - Missing. ⁴⁹⁷ Pa - Missing replaces ⁴⁹⁸ Pa - Missing. ⁴⁹⁹ Pa - Missing replaces ⁵⁰⁰ Pa - 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הליטרא; וערך הסם השני⁵⁶⁵ ה' פשו⁵⁶⁶; וערך הסם⁵⁶⁷ השלישי⁵⁶⁸ ח' פשו⁵⁶⁹; וערך⁵⁷⁰ הרביעי⁵⁷¹ י'א פשו⁵⁷²; וערך⁵⁷³ החמישי⁵⁷⁴ ט'ו פשו⁵⁷⁵; וערך⁵⁷⁶ השישי⁵⁷⁷ י'ט פשו⁵⁷⁸; וערך⁵⁷⁹ השביעי⁵⁸⁰ כ'ח פשו⁵⁸¹; ורצה הקונה לקנות ליטרא מכלם בט'ו פשו⁵⁸². והנה המוסיפים הם שנים: הסם אשר ערכו⁵⁸³ י'ט, פשו⁵⁸⁴ ה'א פשו⁵⁸⁵ ערכו⁵⁸⁶ כ'ח. פשו⁵⁸⁷ ה'א פשו⁵⁸⁸ ערכו⁵⁸⁹ ג' פשו⁵⁹⁰ ערכו⁵⁹¹ ח' פשו⁵⁹² ואשר ערכו⁵⁹³ י'א פשו⁵⁹⁴ הנה⁵⁹⁵ נשים ג' פשו⁵⁹⁶ גילוי⁵⁹⁷ ל'יט⁵⁹⁸ פשו⁵⁹⁹, וה' פשו⁶⁰⁰ גילוי⁶⁰¹ ל'כ' פשו⁶⁰² ומפני⁶⁰³ שנשארו עוד סמים במחסירים⁶⁰⁴ ולא נשאר מהמוסיפים⁶⁰⁵, נקח⁶⁰⁶ מהמוסיפים⁶⁰⁷ היותר קרוב אל ט'ו שהוא הערך המבוקש⁶⁰⁸, והוא י'ט. ונזווג י'ט עם כל אחד מהנשארים מהמחסירים⁶⁰⁹ כמו שתראה בזאת הצורה⁶¹⁰.

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וכבר ידענו איך⁶¹⁰ נקח ליטרא מכל זוג וזוג מאלו, יהיה ערכה ט'ו פשו⁶¹¹. ולזה, נקח מסם ג'⁶¹², ד'⁶¹³ חלקים מיני⁶¹⁴ בליטרא, ומסם י'ט, י'ב חלקים מיני⁶¹⁵ בליט'. ונקח גם כן: מסם ה', י'ג חלקים מכ'ג בליטרא, ומסם כ'ח, י'⁶¹⁶ חלקים מכ'ג בליטרא. ונקח גם כן: מסם⁶¹⁷ ח'י, ד' חלקים מי'א בליטרא, ומסם י'ט, ז'⁶¹⁸ חלקים מי'א בליטרא. ונקח גם כן: מסם⁶¹⁹ י'א, ד' חלקים מח' בליטרא, ומסם⁶²⁰ י'ט, ד' חלקים מח' בליטרא. הנה⁶²¹ מספר כל אלו החלקים⁶²² הוא נ'ח⁶²³, ונוסיף שני חלקים בעבור ט'ו שאין לו גילוי. והנה⁶²⁴ ס', והם חלקי הליטרא. נקח⁶²⁵ מהם⁶²⁶ מסם⁶²⁷ ג' פשו⁶²⁸ ד' חלקים; ומסם ה', י'ג חלקים⁶²⁹; ומסם ח', ד' חלקים; ומסם י'א, ד' חלקים⁶³⁰; ומסם ט'ו, שני חלקים; ומסם י'ט, כ'ג חלקים; ומסם כ'ח, י'⁶³¹ חלקים. ויהיה ערך הליטרא ט'ו פשו⁶³², והוא המבוקש⁶³³.

⁵⁶⁵ NY - השני replaces והשני - ⁵⁶⁶ Mo30 - הליט' is inserted after פשו'. ⁵⁶⁷ Pm2271 - ⁵⁶⁸ NY - השלישי replaces והשלישי - ⁵⁶⁹ Mo30 - הליט' is inserted after פשו'. ⁵⁷⁰ Missing - ⁵⁷¹ NY - הרביעי replaces והרביעי - ⁵⁷² Mo30 - ⁵⁷³ NY - החמישי replaces והחמישי - ⁵⁷⁴ Mo30, Vi - Missing - ⁵⁷⁵ NY - פשו' is inserted after פשו'. ⁵⁷⁶ NY - השישי replaces והשישי - ⁵⁷⁷ Mo30 - Missing - ⁵⁷⁸ Mo30 - ⁵⁷⁹ NY - השביעי replaces והשביעי - ⁵⁸⁰ NY - ⁵⁸¹ NY - פשו' is inserted after פשו'. ⁵⁸² NY - ⁵⁸³ NY - ⁵⁸⁴ NY - ⁵⁸⁵ NY - ⁵⁸⁶ NY - ⁵⁸⁷ NY - ⁵⁸⁸ NY - ⁵⁸⁹ NY - ⁵⁹⁰ NY - ⁵⁹¹ NY - ⁵⁹² NY - ⁵⁹³ NY - ⁵⁹⁴ NY - ⁵⁹⁵ NY - ⁵⁹⁶ NY - ⁵⁹⁷ NY - ⁵⁹⁸ NY - ⁵⁹⁹ NY - ⁶⁰⁰ NY - ⁶⁰¹ NY - ⁶⁰² NY - ⁶⁰³ NY - ⁶⁰⁴ NY - ⁶⁰⁵ NY - ⁶⁰⁶ NY - ⁶⁰⁷ NY - ⁶⁰⁸ NY - ⁶⁰⁹ NY - ⁶¹⁰ NY - ⁶¹¹ NY - ⁶¹² NY - ⁶¹³ NY - ⁶¹⁴ NY - ⁶¹⁵ NY - ⁶¹⁶ NY - ⁶¹⁷ NY - ⁶¹⁸ NY - ⁶¹⁹ NY - ⁶²⁰ NY - ⁶²¹ NY - ⁶²² NY - ⁶²³ NY - ⁶²⁴ NY - ⁶²⁵ NY - ⁶²⁶ NY - ⁶²⁷ NY - ⁶²⁸ NY - ⁶²⁹ NY - ⁶³⁰ NY - ⁶³¹ NY - ⁶³² NY - ⁶³³ NY - ⁶³⁴ NY - ⁶³⁵ NY - ⁶³⁶ NY - ⁶³⁷ NY - ⁶³⁸ NY - ⁶³⁹ NY - ⁶⁴⁰ NY - ⁶⁴¹ NY - 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והיה זה כן, לפי שהוא מבואר ממה שקדם⁶³⁵ שדי לטי⁶³⁶ מערך ג' עם יב' ליטראות מערך יט', ערכם כמו שטח יו' בטי'⁶³⁷. ובזה⁶³⁸ התבאר⁶³⁹ הענין בכל שני מספרים גיליים מאלו. והוא מבואר⁶⁴¹ שב⁶⁴² ליטראות מערך טו', ערכם גם כן כמו שטח ב' בטי'. וכאשר קבצנו⁶⁴³ הכל, הנה ערך כל אלו הליטראות במספרם מוכה בטי' אשר הוא⁶⁴⁴, כמו שטח ס' בטי'.

ואולם, יחס חלק מס' בליטרא אל הליטרא, כמו יחס ערך חלק מס' בליטרא⁶⁴⁶ אל ערך הליטרא⁶⁴⁷. ויתבאר על צד הבאור הקודם⁶⁴⁸, שיחס ערך כל החלקים מקובצים⁶⁴⁹ אל ערך מקובץ⁶⁵⁰ הליטראות, שהוא כמו שטח⁶⁵¹ ס' בטי', הוא כיחס אחד אל ס'. ולזה יהיה ערך החלקים בכללם טו' פשו', שהוא אחד מס' בשטח ס' בטי'. והקש על זה.

13. שאלה: איש אחד שכר איש אחר⁶⁵² מספר מונח מהממון, שיעשה⁶⁵³ מלאכתו מספר מונח מן הימים⁶⁵⁴. וזאת המלאכה אשר יעשה⁶⁵⁵, ישכור לו בכל יום מאלו הימים⁶⁵⁶ מספר מונח מן האנשים, ינהיג כל אחד מהם⁶⁵⁷ מספר מונח מהבהמות⁶⁵⁸, תשא כל אחת מהן מספר מונח מן המדות⁶⁵⁹, ותלך כל אחת מהן מספר מונח מן הדרך⁶⁶⁰. ושנה השכיר בקצת המספרים⁶⁶¹ או בכלם. כמה שכירתו?

קח המספר⁶⁶² המורכב מכל המספרים שהתנה ושמור. גם קח המספר המורכב⁶⁶³ מכל המספרים שהשלים, וכיחס⁶⁶⁴ השמור אל זה המספר המורכב, כן יחס השכירות שהתנה לתת לו אל מה שהוא חייב לתת לו.

דמיון זה, ששכר ראובן⁶⁶⁵ את שמעון יי⁶⁶⁶ ליטר' שיעשה מלאכתו ט'⁶⁶⁷ ימים. וזאת המלאכה⁶⁶⁸ ישכור לו בכל יום יג' מן האנשים, ינהיג כל אחד מהם שבעה מהבהמות⁶⁶⁹, תשא כל אחד מהם טו' מן המדות⁶⁷⁰, ותלך ו' פרסאות. והוא שכר לו ח'⁶⁷¹ ימים, יז' מן האנשים⁶⁷², ותנהיג כל אחד מהם ששה מהבהמות⁶⁷³, ונשאה כל אחת מהם⁶⁷⁴ יא' מן המדות⁶⁷⁵ והלכה ז' פרסאות.

והנה המספר המורכב מהמספרים שהתנה⁶⁷⁶: שהם ט', יג', ז', טו'⁶⁷⁷, ו' הוא⁶⁷⁸ ע'ג' אלפים ותש"י, והוא השמור. והמספר המורכב מהמספרים שהשלים: שהם ח'⁶⁷⁹, יז', יא', ז' הוא ס"ב אלפים ותת"ל"ב. והנה יחס י' ליט' אל מה שהוא חייב⁶⁸⁰ לו, כיחס השמור אל ס"ב אלפים ותת"ל"ב. ואם תערך י' ליט', שהוא הראשון, על הרביעי שהוא ס"ב אלפים ותת"ל"ב⁶⁸¹, ותחלק

⁶³⁵ NY - ממך שקדם replaces מהקודם - ⁶³⁶ Vi - ליט' is inserted after 'ו' - ⁶³⁷ Mo30 - ⁶³⁸ Je, Mo30, ⁶³⁹ Pa, Pm2271 - ⁶⁴⁰ Je - ⁶⁴¹ NY - ⁶⁴² Je, Pa - ⁶⁴³ NY - ⁶⁴⁴ Pa - ⁶⁴⁵ NY - ⁶⁴⁶ Pa - ⁶⁴⁷ Pa - ⁶⁴⁸ Pa - ⁶⁴⁹ NY - ⁶⁵⁰ NY - ⁶⁵¹ Je - ⁶⁵² Pa, NY - ⁶⁵³ NY - ⁶⁵⁴ NY - ⁶⁵⁵ NY - ⁶⁵⁶ NY - ⁶⁵⁷ NY - ⁶⁵⁸ Mo30, ⁶⁵⁹ Pa - ⁶⁶⁰ Pa - ⁶⁶¹ Vi - ⁶⁶² Pa - ⁶⁶³ Pa - ⁶⁶⁴ Vi, Pm2462 - ⁶⁶⁵ NY - ⁶⁶⁶ Pa - ⁶⁶⁷ Pa - ⁶⁶⁸ NY - ⁶⁶⁹ Mo30 - ⁶⁷⁰ Mo30, ⁶⁷¹ Pa - ⁶⁷² Pa - ⁶⁷³ NY - ⁶⁷⁴ NY - ⁶⁷⁵ NY - ⁶⁷⁶ NY - ⁶⁷⁷ Vi, Pm2462 - ⁶⁷⁸ NY - ⁶⁷⁹ NY - ⁶⁸⁰ Pa - ⁶⁸¹ NY - ⁶⁸² NY - ⁶⁸³ NY - ⁶⁸⁴ NY - ⁶⁸⁵ NY - ⁶⁸⁶ NY - ⁶⁸⁷ NY - ⁶⁸⁸ NY - ⁶⁸⁹ NY - ⁶⁹⁰ NY - ⁶⁹¹ NY - ⁶⁹² NY - ⁶⁹³ NY - ⁶⁹⁴ NY - ⁶⁹⁵ NY - ⁶⁹⁶ NY - ⁶⁹⁷ NY - ⁶⁹⁸ NY - ⁶⁹⁹ NY - ⁷⁰⁰ NY - ⁷⁰¹ NY - ⁷⁰² NY - ⁷⁰³ NY - ⁷⁰⁴ NY - ⁷⁰⁵ NY - ⁷⁰⁶ NY - ⁷⁰⁷ NY - ⁷⁰⁸ NY - ⁷⁰⁹ NY - ⁷¹⁰ NY - ⁷¹¹ NY - ⁷¹² NY - ⁷¹³ NY - ⁷¹⁴ NY - ⁷¹⁵ NY - ⁷¹⁶ NY - ⁷¹⁷ NY - ⁷¹⁸ NY - ⁷¹⁹ NY - ⁷²⁰ NY - ⁷²¹ NY - ⁷²² NY - ⁷²³ NY - ⁷²⁴ NY - ⁷²⁵ NY - ⁷²⁶ NY - ⁷²⁷ NY - ⁷²⁸ NY - ⁷²⁹ NY - ⁷³⁰ NY - ⁷³¹ NY - ⁷³² NY - ⁷³³ NY - ⁷³⁴ NY - ⁷³⁵ NY - 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העולה על השמור, יצאו לך הליטראות וחלקי הליטרא שהוא חייב לו. והם ח' ליט' וחצי, ואלף וז' מאות ופ'ה' חלקים, מס'ב' אלפים ותת'ליב' ⁶⁸² בליט', שהם ⁶⁸³ פשו' וניט' ⁶⁸⁴ אלפים תת'ני חלקים מע'ג' אלפים תש'י ⁶⁸⁵ בפשוט.

והיה זה כן, לפי שיחס מה שהוא חייב לו אל מה שהתנה, הוא כיחס מה שעשה אל מה שהתנה לעשות. ויחס מה שעשה אל מה שהתנה ⁶⁸⁶ לעשות ⁶⁸⁷, הוא מחובר מיחסי המספרים שהתנה ⁶⁸⁸ אל גיליהם שהשלים ⁶⁸⁹. וזה היחס ⁶⁹⁰ המחובר, כבר התבאר שהוא כמו יחס ⁶⁹¹ המספר המורכב מהמספרים ⁶⁹² שהתנה אל המספר המורכב מהמספרים ⁶⁹³ שהשלים. והקש על זה.

ומזה הבאור התבאר ⁶⁹⁴: אם ⁶⁹⁵ התנה אדם לאדם ⁶⁹⁶ למלאת ⁶⁹⁷ ממסחר מה כלי, ארכו מספר מונח, ורחבו מספר מונח, ועמקו מספר מונח ⁶⁹⁸, במספר מה מן הממון ⁶⁹⁹; ומלא לו כלי אחר ⁷⁰⁰ מתחלף הרחקים לכלי אשר התנה ⁷⁰¹; כמה חייב לו? וזה, שכמו יחס המספר המורכב מהשלשה רחקים אשר התנה ⁷⁰², אל המספר המורכב מהשלשה רחקים ⁷⁰³ אשר השלים ⁷⁰⁴, כן יחס הערך שהתנה ⁷⁰⁵ לו ⁷⁰⁶ אל מה שהוא חייב ⁷⁰⁷ לו. וכן תשפוט במה שידמה לזה.

והמשל, שמכר המוכר לקונה ⁷⁰⁸ מלא כלי ⁷⁰⁹ משמן בערך כ' ⁷¹⁰ די'. אך הכלי ⁷¹¹ עשרה מדות, ורחבו ⁷¹² תשעה, ועמקו ⁷¹³ יב'. ומלא לו כלי אחר ⁷¹⁴, ארכו אחד ⁷¹⁵ עשר, ורחבו ⁷¹⁶ ששה ⁷¹⁷, ועמקו ⁷¹⁸ שבעה עשר ⁷¹⁹. והנה ⁷²⁰ המספר המורכב מהרחקים הקודמים, הוא אלף ופ' ⁷²¹, והוא השמור. והמספר המורכב ⁷²² מהרחקים שהשלים, הוא ⁷²³ אלף וקכ"ב. והנה כיחס השמור אל אלף וקכ"ב, כן יחס כ' ⁷²⁴ די' אל ⁷²⁵ מה שהוא חייב לו.

(Ed. 1) 14. ⁷²⁶שאלה: ראובן קנה חלקים מה ⁷²⁷ מהמדה במספר ככה מהדינרין, ומכר חלקים ככה מהמדה במספר ככה מהדינרין ⁷²⁸, והיה הממון ככה. ורצינו לדעת ⁷²⁹ אם הרויח או הפסיד וכמה?

הדרך בזה שתוציא סך דינרי ערך המדה לכל אחד מהערכין. ואחר כן יתבאר ⁷³⁰ לך אם הפסיד או הרויח במדה, וכמה מן הדינר. וכזה ⁷³¹ יתבאר ⁷³² לך כמה סך מדות המסחר ⁷³³. ערוך סך מדות המסחר על מה שהרויח במדה או הפסיד,

⁶⁸² Error - Pm2271. ⁶⁸³ ות'ליב' Vi, Pm2462 - מע'ג' אלפים תש'י should be מס'ב' אלפים ותת'ליב'. ⁶⁸⁴ Pa - Missing: ⁶⁸⁵ וני'ה. ⁶⁸⁶ Pa - Missing: ⁶⁸⁷ וני'ה. ⁶⁸⁸ Pa - Missing: ⁶⁸⁹ וני'ה. ⁶⁹⁰ Pa - Missing: ⁶⁹¹ וני'ה. ⁶⁹² Pa - Missing: ⁶⁹³ וני'ה. ⁶⁹⁴ Pa - Missing: ⁶⁹⁵ וני'ה. ⁶⁹⁶ Pa - Missing: ⁶⁹⁷ וני'ה. ⁶⁹⁸ Pa - Missing: ⁶⁹⁹ וני'ה. ⁷⁰⁰ Pa - Missing: ⁷⁰¹ וני'ה. ⁷⁰² Pa - Missing: ⁷⁰³ וני'ה. ⁷⁰⁴ Pa - Missing: ⁷⁰⁵ וני'ה. ⁷⁰⁶ Pa - Missing: ⁷⁰⁷ וני'ה. ⁷⁰⁸ Pa - Missing: ⁷⁰⁹ וני'ה. ⁷¹⁰ Pa - Missing: ⁷¹¹ וני'ה. ⁷¹² Pa - Missing: ⁷¹³ וני'ה. ⁷¹⁴ Pa - Missing: ⁷¹⁵ וני'ה. ⁷¹⁶ Pa - Missing: ⁷¹⁷ וני'ה. ⁷¹⁸ Pa - Missing: ⁷¹⁹ וני'ה. ⁷²⁰ Pa - Missing: ⁷²¹ וני'ה. ⁷²² Pa - Missing: ⁷²³ וני'ה. ⁷²⁴ Pa - Missing: ⁷²⁵ וני'ה. ⁷²⁶ Pa - Missing: ⁷²⁷ וני'ה. ⁷²⁸ Pa - Missing: ⁷²⁹ וני'ה. ⁷³⁰ Pa - Missing: ⁷³¹ וני'ה. ⁷³² Pa - Missing: ⁷³³ וני'ה.

והעולה הוא סך הדינר' שהפסיד או הרויח.

ואם היתה השאלה בהפך. ר"ל שהרויח או הפסיד מספר ככה מהדינרין, ובאת לדעת⁷³⁴ כמה היה הממון או מדות⁷³⁵ המסחר⁷³⁶? תחלק המספר⁷³⁷ מהדינרין שהרויח בכל המסחר⁷³⁸ או הפסיד, על מה שהרויח⁷³⁹ במדה או הפסיד. והעולה הוא סך מדות המסחר, וערכם לפי מה שקנה הוא הממון.

דמיון: ראובן קנה ב' חמישיות המדה וג' שביעיות, ז' דינר וח' חלקים מ'א' דינר. ומכר ד' תשיעיות⁷⁴⁰, ח' דינרין וג' שביעיות⁷⁴¹ הדינר, והיה הממון מאה דינרין. ורצינו לדעת⁷⁴² אם הפסיד או הרויח וכמה? והנה ערך המדה לפי מה שקנה ט' דינרין וקד' חלקים משי"ט' דינר. וערך המדה לפי מה שמכר יח' דינר וכז'⁷⁴³ חלקים מכ"ח' דינר. ולזה היה סך מדות המסחר י' מדות וכ"א' מאות ונ' חלקים מכ"ט' מאות וע"ה⁷⁴⁴ במדה⁷⁴⁵, שהם⁷⁴⁶ פ"ו חלקים מקי"ט' במדה. והנה הרויח במדה, ט' דינר וה' אלפים ותש"א⁷⁴⁷ חלקים מח' אלפים ת'תק"לב' דינר. ערכנוהו על מדות המסחר והנה הרויח ק"ג⁷⁴⁸ דינר וש"ע' אלפים⁷⁴⁹ ומ' חלקים מאלף וס"ב' אלפים⁷⁵⁰ ות'תק"ח' דינר. והם ג' אלפים וקצ' חלקים מט' אלפים קס"ג⁷⁵¹ דינר.

ואם אמר השואל שהרויח ראובן או הפסיד בזה המסחר, קי⁷⁵³ די. ורצית לדעת⁷⁵⁴, כמה הממון? הנה⁷⁵⁵ תחלק במשלנו זה⁷⁵⁶ ק' דינר על מה שהרויח במדה אחת, שהוא ט' דינר וה' אלפים תש"א' חלקים מח' אלפים⁷⁵⁷ ת'תק"לב' דינר. והעולה תהיינה⁷⁵⁸ מדות המסחר. והם י' מדות ולב' אלפים וש"י חלקים מפ"ו אלפים ופ"ט' חלקים במדה. ואם תערוך מדות המסחר על סך דינ' ⁷⁶⁰ ערך המדה לפי מה שקנה, יהיה העולה בידך הממון. וזה כלו מבואר הסבה עם מה שקדם מהדברים⁷⁶¹.

2) (Ed. 14). 762 שאלה: איש אחד קנה חלק מה או חלקים מה ממדה או ליטרא, במספר מונח מהדינר, ומכר חלקים אחרים מחסירים מהראשון במספר ההוא בעינו מהדינרין, והרויח ככה. כמה הממון?

והמשל, שקנה ראובן ב' חמישיות המדה וג' שביעיות, ב' די. ומכר ד' תשיעיות, ב' די⁷⁶³. והרויח ק' די. ורצינו לדעת כמה הממון?

הדרך בזה, שתקח המורה לכל אלו החלקים⁷⁶⁴, הן⁷⁶⁵ מהמדה הן מהדינרין, אם קרה שיהיו בהם חלקים. והנה המורה לכל אלו החלקים במשלנו הוא ש"ט"ו, והיא המדה. קח ממנו ב' חמישיות וג' שביעיות ויעלה רי"ו⁷⁶⁶. הנה אם כן, בדי' יקנה מאלו החלקים, ק"ח. גם קח ממנו ד' תשיעיות ויעלה קמ"י⁷⁶⁷. הנה אם כן, בדי' יתן ע' מאלו החלקים. ולזה ירויח בדי' ל"ח חלקים. וכבר היה

⁷³⁴ NY - לידע. ⁷³⁵ Pa, Pm2271, NY - Missing. ⁷³⁶ Pa - המספר. ⁷³⁷ Vi, Pm2462, Lo - המסחר. ⁷³⁸ Pa - המסחר. ⁷³⁹ Pm2271 - Missing. ⁷⁴⁰ Pa - תשיעיות. ⁷⁴¹ Pa - המספר. ⁷⁴² Vi, Pm2462, Lo - במדה. ⁷⁴³ Pa - Missing. ⁷⁴⁴ Pa - ונ"י. ⁷⁴⁵ Pa - שביעיות. ⁷⁴⁶ Vi, Pm2462, Lo - הם. ⁷⁴⁷ Vi, Pm2462, Lo - ותש"א. ⁷⁴⁸ NY - ק"נ. ⁷⁴⁹ Vi, Pm2462, Lo - ק"י. ⁷⁵⁰ Pa - חלקים. ⁷⁵¹ Pm2462, Lo - לידע. ⁷⁵² Vi - וס"ג. ⁷⁵³ Vi, Pa, Pm2271, Pm2462, Lo - ה'. ⁷⁵⁴ NY - פ"ו. ⁷⁵⁵ NY - חלקים. ⁷⁵⁶ Pa - במשלנו זה. ⁷⁵⁷ NY - Missing. ⁷⁵⁸ NY - הנה. ⁷⁵⁹ Vi, Pm2271, Pm2462, Lo - דינר. ⁷⁶⁰ NY - Missing. ⁷⁶¹ NY - דינר. ⁷⁶² Vi, Pm2462, NY, Lo - Omit this whole problem. ⁷⁶³ Pa, Pm2271 - replaces במדה בדינר. ⁷⁶⁴ Pa - Contains the problem only up to here, but places this fragment after Problem 17. ⁷⁶⁵ Pm2271 - Contains the problem only up to here, but places this fragment after Problem 17. ⁷⁶⁶ Error - This should be 'א'ס"א. The rest of the problem follows this error consistently. ⁷⁶⁷ Mo30 - ת"מ.

הריוח ק' די', שהוא ז' אלפים מאלו החלקים. הנה ראוי שנקח מספר יהיה יחסו אל ק'ח', כיחס ז' אלפים אל ל'ח', וכבר התבאר דרך לקיחתו. ואם חלקת העולה על ע' תמצא מספר הממון עם מה שהריוח יחד. ואם חלקת העולה על ק'ח', שהוא סך חלקי הדינר לפי מה שקנה, תמצא מספר הממון. וזה מבואר הסבה עם מה שקדם מהדברים.

ואם היתה השאלה בהפך: ר"ל, שהפסיד ק' די' בשקנה ד' תשיעיות המדה ב' די', ומכר ב' חמישיותיה⁷⁶⁸ עם ג' שביעיותיה ב' די'; הדרך אחת, אלה שכאשר תחלוק העולה על ע', יהיה לך מספר הממון, ואם תחלקהו על ק'ח', יהיה לך הנשאר לו אחר ההפסד. ואם שאל כמה המקח, חלק העולה על ש'פ'ה⁷⁶⁹, שהם חלקי המדה⁷⁷⁰, והנה מדות המקח. והקש על זה.

וכן אם לא היה מספר הדי' שוה: ר"ל, שקנה חלקים מה מהמדה בג' די', ומכר חלקים מה ממנה כ'ה די', והריוח או הפסיד ק' די'.⁷⁷¹ תוציא בדרך הקודם החלקים מהמורה שיקנה די', גם החלקים שימכר די', ותמשיך המעשה על האופן הקודם, ויצא לך הדרוש.

ואם היו החלקים אחדים ומספר הדי' מתחלף, משל זה: שיקנה ד' חלקים מז' בליט' עם ה' חלקים מי'א' בה'⁷⁷², ז' די' וח' חלקים מי'א' בדי'; וימכר אלו החלקים, ט' די' וג' חלקים מי'ג' די'; והריוח ק' די'; ושאל שואל: כמה הממון או כמה המקח? הנה הדרך שתקח המורה אל כל החלקים הן מהליט', והוא יהיה חלקי⁷⁷⁴ הליט'. ותדע סך החלקים שיקנה בדי', וסך החלקים שיתן בדי'. ותמשיך הענין על האופן הקודם, הן לדעת מספר הממון, הן לדעת מספר המקח.

ואם ישאל אדם: כמה שעור בעל שעור מה, שב'⁷⁷⁵ שביעיותיו עם ג'⁷⁷⁶ חמישיותיו מוסיפים על ד' תשיעיותיו שעור מה? או מחסירים? הנה הדרך בזה מבוארת בתמונת מ'ט'.

15. (Ed. 1). שאלה: ראובן קנה חלקים ככה מהמדה במספר ככה מהדינרין, והיה⁷⁷⁸ הממון מספר ככה מהדינר⁷⁷⁹. ומכר קצת המסחר חלקים⁷⁸⁰ ככה מהמדה⁷⁸¹ במספר ככה מהדינר⁷⁸²; וקצתו האחר, מכר⁷⁸³ חלקים ככה מהמדה במספר ככה מהדינר⁷⁸⁴. ולא הריוח ולא הפסיד. ורצינו לדעת: כמה מהמסחר⁷⁸⁵ מכר לערך הראשון וכמה לערך השני?

הנה מן המחוייב⁷⁸⁶, אם היתה⁷⁸⁷ השאלה צודקת⁷⁸⁸, שיריוח בערך האחד ויפסיד בערך השני. והנה תוציא ערך⁷⁸⁹ המדה לכל אחד מהערכים, ותראה כמה הפסיד במדה לערך האחד, וכמה הריוח⁷⁹⁰ במדה לערך השני. והנה כיחס ההפסד אל הריוח⁷⁹¹, כן יחס מה שמכר לערך המותיר אל מה שמכר לערך המחסיר⁷⁹². וזה יתבאר⁷⁹³ במעט עיון⁷⁹⁴ מתמונת ל'ט' מהמאמר⁷⁹⁵ הראשון.

⁷⁶⁸ Mo30 - Missing. ⁷⁶⁹ Error - Should be 'ש'ט'ו'. ⁷⁷⁰ Je - המורה. ⁷⁷¹ Mo30 - is inserted before ⁷⁷² Je - Missing. ⁷⁷³ Error - This should be ⁷⁷⁴ Je - Missing. ⁷⁷⁵ Mo1063 - חלק. ⁷⁷⁶ Mo1063 - יח'. ⁷⁷⁷ Je, Mo30, Mo1063 - Omit this whole problem. ⁷⁷⁸ Vi, Pm2462 - והוא. ⁷⁷⁹ Pm2271 - מן הדינר. ⁷⁸⁰ Pa - Missing. ⁷⁸¹ Pm2462, Lo - מן המדה. ⁷⁸² Pa - מן הדינר. ⁷⁸³ Pm2271, NY - Missing. ⁷⁸⁴ Pa - מן הדינר. ⁷⁸⁵ Vi - מהמספר. ⁷⁸⁶ NY - Missing. ⁷⁸⁷ NY - מן המחוייב. ⁷⁸⁸ NY - Missing. ⁷⁸⁹ NY - Missing. ⁷⁹⁰ NY - Missing. ⁷⁹¹ Lo - הפסיד. ⁷⁹² NY - Missing. ⁷⁹³ NY - Missing. ⁷⁹⁴ NY - Missing. ⁷⁹⁵ Vi, Pm2462, Lo - מן. ⁷⁹⁶ NY - Missing. ⁷⁹⁷ NY - Missing. ⁷⁹⁸ NY - Missing. ⁷⁹⁹ NY - Missing. ⁸⁰⁰ NY - Missing. ⁸⁰¹ NY - Missing. ⁸⁰² NY - Missing. ⁸⁰³ NY - Missing. ⁸⁰⁴ NY - Missing. ⁸⁰⁵ NY - Missing. ⁸⁰⁶ NY - Missing. ⁸⁰⁷ NY - Missing. ⁸⁰⁸ NY - Missing. ⁸⁰⁹ NY - Missing. ⁸¹⁰ NY - Missing. ⁸¹¹ NY - Missing. ⁸¹² NY - Missing. ⁸¹³ NY - Missing. ⁸¹⁴ NY - Missing. ⁸¹⁵ NY - Missing. ⁸¹⁶ NY - Missing. ⁸¹⁷ NY - Missing. ⁸¹⁸ NY - Missing. ⁸¹⁹ NY - Missing. ⁸²⁰ NY - Missing. ⁸²¹ NY - Missing. ⁸²² NY - Missing. ⁸²³ NY - Missing. ⁸²⁴ NY - Missing. ⁸²⁵ NY - Missing. ⁸²⁶ NY - Missing. ⁸²⁷ NY - Missing. ⁸²⁸ NY - Missing. ⁸²⁹ NY - Missing. ⁸³⁰ NY - Missing. ⁸³¹ NY - Missing. ⁸³² NY - Missing. ⁸³³ NY - Missing. ⁸³⁴ NY - Missing. ⁸³⁵ NY - Missing. ⁸³⁶ NY - Missing. ⁸³⁷ NY - Missing. ⁸³⁸ NY - Missing. ⁸³⁹ NY - Missing. ⁸⁴⁰ NY - Missing. ⁸⁴¹ NY - Missing. ⁸⁴² NY - Missing. ⁸⁴³ NY - Missing. ⁸⁴⁴ NY - Missing. ⁸⁴⁵ NY - Missing. ⁸⁴⁶ NY - Missing. ⁸⁴⁷ NY - Missing. ⁸⁴⁸ NY - Missing. ⁸⁴⁹ NY - Missing. ⁸⁵⁰ NY - Missing. ⁸⁵¹ NY - Missing. ⁸⁵² NY - Missing. ⁸⁵³ NY - Missing. ⁸⁵⁴ NY - Missing. ⁸⁵⁵ NY - Missing. ⁸⁵⁶ NY - Missing. ⁸⁵⁷ NY - Missing. ⁸⁵⁸ NY - Missing. ⁸⁵⁹ NY - Missing. ⁸⁶⁰ NY - Missing. ⁸⁶¹ NY - Missing. ⁸⁶² NY - Missing. ⁸⁶³ NY - Missing. ⁸⁶⁴ NY - Missing. ⁸⁶⁵ NY - Missing. ⁸⁶⁶ NY - Missing. ⁸⁶⁷ NY - Missing. ⁸⁶⁸ NY - Missing. ⁸⁶⁹ NY - Missing. ⁸⁷⁰ NY - Missing. ⁸⁷¹ NY - Missing. ⁸⁷² NY - Missing. ⁸⁷³ NY - Missing. ⁸⁷⁴ NY - Missing. ⁸⁷⁵ NY - Missing. ⁸⁷⁶ NY - Missing. ⁸⁷⁷ NY - Missing. ⁸⁷⁸ NY - Missing. ⁸⁷⁹ NY - Missing. ⁸⁸⁰ NY - Missing. ⁸⁸¹ NY - Missing. ⁸⁸² NY - Missing. ⁸⁸³ NY - Missing. ⁸⁸⁴ NY - Missing. ⁸⁸⁵ NY - Missing. ⁸⁸⁶ NY - Missing. ⁸⁸⁷ NY - Missing. ⁸⁸⁸ NY - Missing. ⁸⁸⁹ NY - Missing. ⁸⁹⁰ NY - Missing. ⁸⁹¹ NY - Missing. ⁸⁹² NY - Missing. ⁸⁹³ NY - Missing. ⁸⁹⁴ NY - Missing. ⁸⁹⁵ NY - Missing. ⁸⁹⁶ NY - Missing. ⁸⁹⁷ NY - Missing. ⁸⁹⁸ NY - Missing. ⁸⁹⁹ NY - Missing. ⁹⁰⁰ NY - Missing. ⁹⁰¹ NY - Missing. ⁹⁰² NY - Missing. ⁹⁰³ NY - Missing. ⁹⁰⁴ NY - Missing. ⁹⁰⁵ NY - Missing. ⁹⁰⁶ NY - Missing. ⁹⁰⁷ NY - Missing. ⁹⁰⁸ NY - Missing. ⁹⁰⁹ NY - Missing. ⁹¹⁰ NY - Missing. ⁹¹¹ NY - Missing. ⁹¹² NY - Missing. ⁹¹³ NY - Missing. ⁹¹⁴ NY - Missing. ⁹¹⁵ NY - Missing. ⁹¹⁶ NY - Missing. ⁹¹⁷ NY - Missing. ⁹¹⁸ NY - Missing. ⁹¹⁹ NY - Missing. ⁹²⁰ NY - Missing. ⁹²¹ NY - Missing. ⁹²² NY - Missing. ⁹²³ NY - Missing. ⁹²⁴ NY - Missing. ⁹²⁵ NY - Missing. ⁹²⁶ NY - Missing. ⁹²⁷ NY - Missing. ⁹²⁸ NY - Missing. ⁹²⁹ NY - Missing. ⁹³⁰ NY - Missing. ⁹³¹ NY - Missing. ⁹³² NY - Missing. ⁹³³ NY - Missing. ⁹³⁴ NY - Missing. ⁹³⁵ NY - Missing. ⁹³⁶ NY - Missing. ⁹³⁷ NY - Missing. ⁹³⁸ NY - Missing. ⁹³⁹ NY - Missing. ⁹⁴⁰ NY - Missing. ⁹⁴¹ NY - Missing. ⁹⁴² NY - Missing. ⁹⁴³ NY - Missing. ⁹⁴⁴ NY - Missing. ⁹⁴⁵ NY - Missing. ⁹⁴⁶ NY - Missing. ⁹⁴⁷ NY - Missing. ⁹⁴⁸ NY - Missing. ⁹⁴⁹ NY - Missing. ⁹⁵⁰ NY - Missing. ⁹⁵¹ NY - Missing. ⁹⁵² NY - Missing. ⁹⁵³ NY - Missing. ⁹⁵⁴ NY - Missing. ⁹⁵⁵ NY - Missing. ⁹⁵⁶ NY - Missing. ⁹⁵⁷ NY - Missing. ⁹⁵⁸ NY - Missing. ⁹⁵⁹ NY - Missing. ⁹⁶⁰ NY - Missing. ⁹⁶¹ NY - Missing. ⁹⁶² NY - Missing. ⁹⁶³ NY - Missing. ⁹⁶⁴ NY - Missing. ⁹⁶⁵ NY - Missing. ⁹⁶⁶ NY - Missing. ⁹⁶⁷ NY - Missing. ⁹⁶⁸ NY - Missing. ⁹⁶⁹ NY - Missing. ⁹⁷⁰ NY - Missing. ⁹⁷¹ NY - Missing. ⁹⁷² NY - Missing. ⁹⁷³ NY - Missing. ⁹⁷⁴ NY - Missing. ⁹⁷⁵ NY - Missing. ⁹⁷⁶ NY - Missing. ⁹⁷⁷ NY - Missing. ⁹⁷⁸ NY - Missing. ⁹⁷⁹ NY - Missing. ⁹⁸⁰ NY - Missing. ⁹⁸¹ NY - Missing. ⁹⁸² NY - Missing. ⁹⁸³ NY - Missing. ⁹⁸⁴ NY - Missing. ⁹⁸⁵ NY - Missing. ⁹⁸⁶ NY - Missing. ⁹⁸⁷ NY - Missing. ⁹⁸⁸ NY - Missing. ⁹⁸⁹ NY - Missing. ⁹⁹⁰ NY - Missing. ⁹⁹¹ NY - Missing. ⁹⁹² NY - Missing. ⁹⁹³ NY - Missing. ⁹⁹⁴ NY - Missing. ⁹⁹⁵ NY - Missing. ⁹⁹⁶ NY - Missing. ⁹⁹⁷ NY - Missing. ⁹⁹⁸ NY - Missing. ⁹⁹⁹ NY - Missing. ¹⁰⁰⁰ NY - Missing.

וכאשר קבצנו, יהיה יחס ההפסד אל הריוח וההפסד מקובצים⁷⁹⁶, כיחס מה שמכר לערך⁷⁹⁷ המותיר אל כל המסחר. ולפי ששלשה מאלו ידועים לך⁷⁹⁸, תוכל להוציא הרביעי על האופן⁷⁹⁹ הקודם⁸⁰⁰ בראש זה השער⁸⁰¹.

דמיון זה⁸⁰², ראובן קנה ב' שביעיות המדה, ג' דינרין ורביע דינר⁸⁰³, והיה הממון ק' דינר. ומכר קצת המסחר, ד' תשיעיות המדה, ו' דינרין ושליש. וקצתו, מכר ה' חלקים מ'א' במדה, ד' דינר וחומש דינר. והנה ערך המדה לפי מה שקנה, י'א' דינר וג' שמיניות הדינר. וערכה לפי⁸⁰⁴ הערך⁸⁰⁵ הראשון⁸⁰⁶, י'ד' דינר ורביע דינר. ולפי הערך⁸⁰⁷ השני⁸⁰⁸, יהיה ערכה ט' דינר וו' חלקים מ'ה' בדינר. ולזה יהיה המסחר, ח' מדות וע'ב' חלקים מ'צ'א' בדינר. והנה מה שהרויח במדה לפי הערך⁸⁰⁹ הראשון⁸¹⁰ הם ב' דינר וז' שמיניות הדינר⁸¹¹. ומה שהפסיד במדה לפי הערך⁸¹² השני⁸¹³ הם ב' דינר וכ'ז' חלקים ממאתים בדינר. והנה ההפסד⁸¹⁴ והרויח מקובצים⁸¹⁵ הם ה' דינר וב' חלקים ממאתים בדינר.

ערוך סך דינרי ההפסד ושבריהם על סך מדות המסחר⁸¹⁶ ושבריהן, וחלק העולה על סך דינרי ההפסד והריוח מקובצים⁸¹⁷ ושבריהם. והעולה הוא⁸¹⁸ סך מדות המסחר שמכר לערך המותיר ושבריהם. ערכת ב' שלמים וכ'ז' חלקים ממאתים באחד, על ח' שלמים וע'ב' חלקים מ'צ'א' באחד. וחלקת העולה על ה' שלמים וב' חלקים ממאתים באחד, והעולה מן החלוקה⁸²⁰ הוא סך⁸²¹ המדות⁸²² שמכר לערך המותיר ושבריהן. והנשאר מן המסחר⁸²³, מכר לערך המחסיר⁸²⁴.

ואם אמר השואל שהרויח או הפסיד מספר מה מן הדינרין⁸²⁵, תקח מספר המדות אשר ירויח⁸²⁶ בהם⁸²⁷ זה הסך אם הרויח, או אשר יפסיד בהם⁸²⁸ זה הסך⁸²⁹ אם הפסיד, ותוציא⁸³⁰ מן המסחר⁸³¹ ושמהו. והנשאר⁸³³ מהמסחר⁸³⁴ תחלקו⁸³⁵ על האופן הקודם, ותמצא מה שמכר לערך המותיר ממנו. והשאר⁸³⁶ ממנו⁸³⁷ הוא מה שמכר לערך המחסיר⁸³⁸. ואם הפסיד, תחבר⁸³⁹ מה שמכר לערך המחסיר⁸⁴⁰ עם השמור. ואם הרויח, תחברו⁸⁴¹ עם מה שמכר לערך המותיר.

ואם היתה השאלה⁸⁴² שמכר כ' מדות⁸⁴³ לערך המותיר, ורצינו לדעת⁸⁴⁴ כמה מן המדות⁸⁴⁵ מכר לערך המחסיר⁸⁴⁶ בדרך⁸⁴⁷ שלא ירויח ולא יפסיד; הנה כיחס כ' מדות אל הנעלם, כן יחס ב' דינרין וכ'ז' חלקים ממאתים בדינר אל כ' דינר וז' שמיניות הדינר. ולזה⁸⁴⁸ תוכל לדעת הנעלם על האופן שקדם⁸⁴⁹.

⁷⁹⁶ Pa, על האופן replaces בדרך - NY ⁷⁹⁹. לך. NY - Missing ⁷⁹⁸. לחלק. NY ⁷⁹⁷. נקבצים. NY ⁷⁹⁶.
⁸⁰⁰ Pa, זה. NY, Lo - Missing ⁸⁰². זה השער replaces שער זה - NY ⁸⁰¹. שקדם - NY, Pm2271, NY ⁸⁰³.
⁸⁰⁵ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸⁰⁶. ראשון - NY ⁸⁰⁷. שני - NY ⁸⁰⁸.
⁸¹⁰ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹³ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁴ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁵ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁶ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁷ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁸ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸¹⁹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁰ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²¹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²² Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²³ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁴ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁵ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁶ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁷ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁸ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸²⁹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁰ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³¹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³² Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³³ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁴ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁵ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁶ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁷ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁸ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸³⁹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁰ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴¹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴² Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴³ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁴ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁵ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁶ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁷ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁸ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.
⁸⁴⁹ Pa, Pm2271, NY, Lo - Missing ⁸⁰². ערך. NY ⁸¹². ה' דינר - NY ⁸¹¹. ראשון - NY ⁸¹⁰. ערך. NY ⁸⁰⁹.

וממה שקדם יתבאר לך: אם החליף אדם⁸⁵⁰ מסחר⁸⁵¹ מה במסחר אחר⁸⁵²; וערך החלקים⁸⁵³ מן המדה⁸⁵⁴ ממה שנתן, מספר ככה מן הדינר; וערך חלקים אחרים⁸⁵⁵ מן המסחר⁸⁵⁶ שקבל ככה; ונתן מספר⁸⁵⁷ ככה מן המדות⁸⁵⁸; כמה ראוי שיקבל? וזה שתוציא ערך המדה לכל אחד מהערכים. וכיחס ערך המדה שנתן⁸⁵⁹ אל ערך⁸⁶⁰ המדה שקבל, כן יחס סך המדות שקבל אל סך⁸⁶¹ המדות שנתן. ושלושה מאלו המספרים ידועים לך⁸⁶², ולזה תוכל להוציא הרביעי.

וכן⁸⁶³ אם ידעת סך המדות שנתן ושקבל מקובצות, הנה⁸⁶⁴ תוכל להוציא כמה מן המדות⁸⁶⁵ נתן, וכמה קבל. וזה שתוציא ערך המדה לכל אחד מהערכים, וכיחס ערך המדה ממה שנתן אל מקובץ⁸⁶⁶ ערכי המדה ממה שנתן ושקבל, כן יחס סך⁸⁶⁷ המדות שקבל אל מקובץ⁸⁶⁸ סך המדות שנתן ושקבל. והנה שלושה⁸⁶⁹ מאלו המספרים⁸⁷⁰ ידועים לך⁸⁷¹ ומהם⁸⁷² תוכל⁸⁷³ להוציא⁸⁷⁴ הרביעי.

וכן יתבאר⁸⁷⁵ לך: אם החליף אדם מסחר⁸⁷⁶ מה בשני מסחרים, וערך חלקים מה מן המדה⁸⁷⁷ ממה שנתן מספר ככה מן הדינר; וערך חלקים אחרים מן המדה⁸⁷⁸ מאחד מן המסחרים שקבל ככה; וערך חלקים אחרים מן המדה⁸⁷⁹ משני המסחרים⁸⁸⁰ שקבל ככה; ונתן מספר ככה מהמדות⁸⁸²; וקבל מספר ככה מהמדות בין שני המסחרים. ורצית לדעת⁸⁸³ כמה מן המדות⁸⁸⁴ קבל מהמסחר⁸⁸⁵ האחד⁸⁸⁶, וכמה⁸⁸⁷ מן המסחר⁸⁸⁸ השני?

הנה תוציא ערך מה שנתן בכללו, ותחלקהו על מספר המדות שקבל. והוא יהיה ערך המדה המעורבת משני אלו המסחרים⁸⁸⁹. וכבר קדמה לך הדרך, כמה חלקים מהמדה ילקחו⁸⁹⁰ מהמסחר האחד⁸⁹¹ וכמה⁸⁹² מהמסחר⁸⁹³ השני, ויהיה ערך המדה ככה. ולזה תכפיל⁸⁹⁴ מספר חלקי המדה שיקח מהמספר⁸⁹⁵ האחד⁸⁹⁶, על מספר מדות⁸⁹⁷ השני⁸⁹⁸ מסחרים⁸⁹⁹ שקבל. והחלקים ההם, הם חלקי המדה שקבל מזה המסחר, והשאר קבל מהמסחר השני. וסבת זה מבוארת עם מה שקדם מהדברים⁹⁰⁰.

דמיון, ראובן נתן לשמעון ז' מדות ממסחר מה⁹⁰¹, ערך המדה ב' דינר ושיעית; וקבל משמעון ט' מדות משני מסחרים⁹⁰², ערך המדה מהמסחר⁹⁰³ האחד, דינר וחצי, וערך המדה מהמסחר⁹⁰⁴ השני⁹⁰⁵ ב' דינר וג' רביעיות. ויהיה ערך⁹⁰⁶ ה' מדות שקבל כערך⁹⁰⁷ הז' מדות שנתן. ורצינו לדעת⁹⁰⁸: כמה מדות⁹⁰⁹ קבל

מהמדה - NY⁸⁵⁴. חלקים - Pm2462, Lo⁸⁵³. אחד - Pm2271⁸⁵². מספר - Pa⁸⁵¹. אדם - Pa⁸⁵⁰ Missing. מן המדות replaces מהמדות - NY⁸⁵⁵. המספר - Pa⁸⁵⁶. מהמסחר - NY⁸⁵⁶. אחרים - NY⁸⁵⁵ Missing. כן - Pa⁸⁶³. לך - NY⁸⁶² Missing. ערך - Pa⁸⁶¹. לערך - NY⁸⁶⁰. שנתן - NY⁸⁵⁹ Missing. סך - Vi, Pm2462, Lo⁸⁶⁷ Missing. נקבץ - NY⁸⁶⁶. מן המדות replaces מהמדות - Pa, NY⁸⁶⁵. והנה replaces וג' ממספרים אלו - NY⁸⁷⁰. השלושה - Lo⁸⁶⁹. אל מקובץ replaces לנקבץ - NY⁸⁶⁸. תוציא - Pm2462⁸⁷³. וליה - NY⁸⁷². ויהיה - Pa⁸⁷². לך - NY⁸⁷¹ Missing. שלושה מאלו המספרים - Pa, Pm2271⁸⁷⁸. מן המדה replaces מהמדה - NY⁸⁷⁷. מספר - Pa⁸⁷⁶. יבואר - NY⁸⁷⁵. לידע - NY⁸⁷⁴. מאחד - Pm2271, NY⁸⁷⁹ Missing. מן המדה replaces מהמדה - Pa⁸⁷⁹. מן המדה replaces מהמדה - NY⁸⁸⁰. מהשני - NY⁸⁸¹ Pm2271, NY⁸⁸⁰. מן המסחרים שקבל ככה; וערך חלקים אחרים מן המדה - NY⁸⁸² Pa⁸⁸². מן המדות - Pa⁸⁸². מהמסחרים - NY⁸⁸⁵ Vi⁸⁸⁵. האחר - Pm2271⁸⁸⁶. המספר - NY⁸⁸⁷ Pm2271, NY⁸⁸⁶. מן המסחר - Pa⁸⁸⁹. מהמספר - Pa, Pm2462, Lo⁸⁸⁸. וככה - NY⁸⁹¹ Pm2271. לקחו - Pa⁸⁹⁰. מסחרים אלו - NY⁸⁸⁹. מן המסחר replaces מהמסחר - Pa, NY⁸⁸⁸. וככה - Vi⁸⁹⁵. כפול - NY⁸⁹⁴. מן המסחר - Pm2271⁸⁹³. וככה - Vi, NY⁸⁹². האחד - Pa⁸⁹² Missing. האחר - NY⁸⁹⁸ Pa, Pm2462, Vi⁸⁹⁷. המדות - NY⁸⁹⁶ Pm2271. מהמסחר - NY⁸⁹⁶. מה מסדר - NY⁸⁹⁹ Pm2271, NY⁹⁰¹ Missing. עם מה שקדם מהדברים replaces מהקודם - NY⁹⁰⁰. המסחרים - NY⁹⁰⁰ Pm2271, NY⁹⁰⁰. הערך - Vi⁹⁰⁶. שני - NY⁹⁰⁵. ממסחר - NY⁹⁰⁴. ממסחר - NY⁹⁰³. מהמסחרים - Vi⁹⁰². מה מהמדות - NY⁹⁰⁹ Pa, Pm2271. לידע - NY⁹⁰⁸. בערך - Pm2462, Pm2271, Lo⁹⁰⁷.

מהמסחר⁹¹⁰ האחד, וכמה⁹¹¹ מהמסחר⁹¹² השני:

הנה ידענו שערך מה שקבל הם ט'ו' דינר. ולזה יהיה ערך המדה ממה שקבל, דינר וב' שלישיות. ולפי שהיה ערך המדה מהמסחר⁹¹³ האחד דינר וחצי ומהמסחר השני⁹¹⁴ ב' דינרין וג' רביעיות⁹¹⁵, הנה יקח ב' חלקים מט'ו' במדה מהמסחר אשר ערך⁹¹⁶ המדה ממנו ב' דינ' וג' רביעיות, ויג' חלקים מהמדה מהמסחר האחר. ויהיה⁹¹⁷ ערך המדה, דינר וב' שלישיות הדינר. וזה יתבאר כשהשיבנו כל שברי הדינרין אל חלקים ממין אחד⁹¹⁸. רצוני⁹¹⁹, השליש והחצי והרביע. וזה שחסרון המחסיר⁹²⁰ הוא שנים מאלו החלקים⁹²¹, ויתרון המוסיף הוא יג' מאלו החלקים⁹²². ובהיות הענין כן הוא מבואר⁹²³, שהוא יקח מהמסחר אשר ערך⁹²⁴ המדה⁹²⁵ ממנו דינר וחצי, ז' מדות וד' חומשי המדה; והשאר יקח מהמסחר האחר.

וכזה⁹²⁶ התבאר⁹²⁷ לך, אם התנה אדם לתת בשכירות⁹²⁸ לאיש מה בזמן מה מספר ככה מהדינרין, ובכל יום שיבטל, הפועל יתן לשוכר⁹²⁹ מספר ככה מהדינ', ועשה הפועל מלאכתו מספר מונה מהימים, ובטל זמן מה, ויצא שכרו בהפסדו, כמה מהזמן בטל? וזה, שפוציא הסך מהדי' שירויח הפועל ביום כשיעשה מלאכתו, והסך מהדי' שיפסיד ביום שיבטל. וכיחס סך דינר ההפסד אל סך דינר הרויח, כן יחס הזמן שבטל אל הזמן שעשה⁹³⁰. וזה מבואר בנפשו⁹³¹. והשלשה מאלו המספרים⁹³² ידועים, ומהם תוציא הרביעי. וכן אם היה⁹³³ ידוע לך⁹³⁴ מקובץ⁹³⁵ שני הזמנים, תוכל לדעת⁹³⁶ באופן⁹³⁷ הקודם כמה מן הזמן⁹³⁸ בטל, וכמה עשה.

15. (Ed. 2). ⁹³⁹שאלה: איש⁹⁴⁰ אחד החליף⁹⁴¹ מסחר מה במסחר אחר. וערך חלקים מה מן המדה ממה שנתן, ככה. וערך חלקים אחרים מהמסחר שקבל, ככה. ונתן מספר ככה מהמדות. כמה ראוי שיקבל?

והמשל, שהמסחר שנתן, ישו ג' חמישיותיו המדה עם ב' תשיעיותיו, ב' די'. והמסחר שקבל, ישו ג' חלקים מי'א' במדה עם ב' שביעיותיה, ג' די'. ונתן לו כ' מדות, ורצינו לדעת כמה ראוי שיקבל?

הדרך בזה שתקח המורה אל כל החלקים, הן מהמדה הן מהדינרין, אם⁹⁴² קרה שיהיו שם חלקים. וממנו תקח החלקים שילקחו בדי' לכל אחד מאלו הערכים. וכמו חלקי הדי' ממה שנתן אל חלקי הדי' ממה שקבל, כן יחס מה שנתן אל מה שקבל. והנה המורה לכל אלו החלקים הוא ג' אלפים ת'ס'ה'. לקחנו ג' חמישיותיו וב' תשיעיותיו והנה אלפים ת'ס'ד'⁹⁴³. ולזה יהיו החלקים שילקחו בדינר ממה שנתן, אלף ר'ל'ב'. לקחנו ב' שביעיותיו וג' חלקים מי'א' במדה בו,

⁹¹⁰ Pm2462, Lo - מהמסחר. ⁹¹¹ Pm2271, NY - וככה. Vi - is inserted after קבל. ⁹¹² NY - שביעיות. ⁹¹³ Pm2271, NY - ומהמסחר השני replaces מהשני. ⁹¹⁴ NY - ממסחר. ⁹¹⁵ NY - ממסחר. ⁹¹⁶ NY - ר"ל. ⁹¹⁷ NY - ממין אחד. ⁹¹⁸ Pa - Missing. ⁹¹⁹ NY - והיה. ⁹²⁰ NY - אשר ערך replaces שערך. ⁹²¹ NY - מאלו replaces מחלקים אלו. ⁹²² NY - מאלו החלקים replaces מחלקים אלו. ⁹²³ NY - החוסר. ⁹²⁴ NY - ובהיות הענין כן הוא מבואר replaces וא"כ ידוע. ⁹²⁵ NY - החלקים. ⁹²⁶ NY - לשוחר. ⁹²⁷ NY - בשכרו. ⁹²⁸ Vi - יתבאר. ⁹²⁹ NY - ובזה. ⁹³⁰ Pm2462, NY - מדה. ⁹³¹ Pm2271, NY - ערך. ⁹³² NY - וזה מבואר בנפשו. ⁹³³ NY - Missing. ⁹³⁴ NY - הזמן שיעשה מלאכתו. ⁹³⁵ Pa - לך. ⁹³⁶ Pm2271, NY - היה. ⁹³⁷ Pa - Missing. ⁹³⁸ NY - והשלשה מאלו המספרים replaces וג' ממספרים אלו. ⁹³⁹ NY - מקובץ replaces מהזמן. ⁹⁴⁰ Pa - בצד. ⁹⁴¹ NY - לידע. ⁹⁴² NY - נקבץ. ⁹⁴³ NY - ידוע לך replaces ידוע. ⁹⁴⁴ NY - Error - This should be 'ת'ת'מ'ט'. The rest of the problem follows this error consistently. ⁹⁴⁵ Mo30 - וראם. ⁹⁴⁶ Mo30 - מחליף. ⁹⁴⁷ Mo30 - הזמן.

ועלה אלף ת'ת'ק'ל'ה'. ולזה יהיו החלקים שילקחו בדי' ממה שקבל, ת'ר'מ'ה'.
והנה כיחס אלף ר'ל'ב' אל ת'ר'מ'ה', כן יחס מה שקבל אל כ' מדות. וכבר ידעת
דרך לקיחת זה המספר הנעלם. וסבת זה מבוארת. והקש על זה.

ומזה התבאר לך: אם התנה אדם לתת בשכירות לאיש מה, בזמן מה, מספר ככה
מהדינ'. ובכל יום שיבטל הפועל, יתן לשוכר מספר ככה מהדינ'. ועשה הפועל
זמן מה ובטל, ויצא שכרו בהפסדו. כמה בטל?

משל זה, שראובן שכר שמעון לעשות מלאכתו, ויתן לו בשכירותו בז' ימים
וחומש יום, כ' די'. ושמעון יתן לראובן בכל יום שיבטל, ד' חלקים מי'א' בדי'.
ועשה שמעון ב' ⁹⁴⁵ימים וג' שביעיות יום ובטל, ויצא שכרו בהפסדו. ורצינו
לדעת: כמה הבטול? ⁹⁴⁶

לקחנו המורה לכל החלקים בכללם, ועלה ש'פ'ה' והוא חלקי היום. הנה נראה
בכמה חלקים מאלו ⁹⁴⁷ירויה די', ובכמה חלקים מאלו יפסידהו. וכיחס מספר
חלקי הירויה אל מספר חלקי ההפסד, כן יחס ב' ⁹⁴⁸ימים וג' שביעיות ⁹⁴⁹אל
הזמן שבטל. והנה בק'ל'ח' חלקים וי'ב' חלקים מעשרים בחלק, ירויה די'.
ובאלף ונ'ח' ⁹⁵⁰חלקים וג' חלקים מארבעה בחלק, יפסיד די'. וככה ⁹⁵¹יחס ב' ⁹⁵²
ימים וג' שביעיות אל הזמן שבטל. והקש על זה.

ואם שאל לך אדם: כמה המספר שיהיו חלקים מה ממנו כמו חלקים מונחים
ממספר מונח? הנה הדרך בזה התבארה בתמונת נ'א'.

APPENDIX

List of the Theorems from Part One of *Maaseh Hoshev* in the Two Editions,
with Brief Notes

The regular numbering is from the first edition, and the numbering in parentheses is from
the second.

- 1. The product of two numbers a and b , is added to itself b times. (No proof).
- (1) 2. The product of a number a and another $b = b_1 + b_2 + \dots + b_n$ is $ab_1 + ab_2 + \dots + ab_n$. (Proof just unravels the theorem using 1.)
- (2) 3. The product of two numbers $a = a_1 + a_2 + \dots + a_m$ and $b = b_1 + b_2 + \dots + b_n$ is $a_1b_1 + a_1b_2 + \dots + a_1b_n + a_2b_1 + a_2b_2 + \dots + a_2b_n + \dots + a_mb_1 + a_mb_2 + \dots + a_mb_n$. (Proof just unravels the theorem using 2.)
- (3) 4. The product of a number $a = b + c$ with b is equal to $b^2 + bc$. (Corollary of 2, proof is immediate.)
- (4) 5. $(a/2 + b)$ squared is equal to $(a + b)b + (a/2)^2$. (Proof uses 3.)
- (5) 6. $(a + b)^2 = a^2 + b^2 + 2ab$. (Proof uses 3.)
- (6) 7. $(a + b)^2 = a(a + b) + ab + b^2$. (Proof uses 3.)
- 8. If $a = b + c$, then either $(a/2)^2 = bc + (b - a/2)^2$ or $(a/2)^2 = bc + (c - a/2)^2$. (Proof uses 3. Does not consider the negative case.)
- (7) 9. $a(bc) = b(ac) = c(ab)$. (Proof uses 1.)

⁹⁴⁵ Mo30 - 'כ'. ⁹⁴⁶ בטל Mo30. ⁹⁴⁷ Mo1063 - יהיה is inserted after אלף. ⁹⁴⁸ Mo30, Je -
⁹⁴⁹ 'כ'. ⁹⁵⁰ חמישיות - Mo30. ⁹⁵¹ 'יח'. ⁹⁵² Je - וככה is inserted after חלק Mo30.

- (8) 10. $a(bcd) = b(acd) = c(abd) = d(abc)$. (Proof uses induction and 9, and explicitly implies a more general theorem, that you can take any n numbers, and their product will be the same as the product of any $n - 1$ terms times the remaining term.)
- (9) 11. $a(bcd) = (ac)(bd)$. (Proof uses 10, and explicitly implies a more general theorem, that you can take any n numbers, and their product will be the same as the product of any $n - 2$ terms times the product of the remaining two terms.)
- (10) 12. $a(b_1)(b_2) \cdots (b_n) = b_1(a)(b_2)(b_3) \cdots (b_n) = b_2(a)(b_1)(b_3) \cdots (b_n) = \cdots = b_n(a)(b_1)(b_2) \cdots (b_{n-1})$. (Completely generalizes 10 and 11.)
- (11) 13. The ratio $((a_1)(a_2) \cdots (a_n))/((b_1)(b_2) \cdots (b_n)) = (a_1/b_1)(a_2/b_2) \cdots (a_n/b_n)$.
- (12) 14. The ratio $((a_1)(a_2) \cdots (a_n))/((b_1)(b_2) \cdots (b_n))$ equals the product of a_i/b_j , where each i and j between 1 and n appears exactly once.
- 15. If a is relatively prime to $b = (b_1)(b_2) \cdots (b_n)$, then a is relatively prime to b_i , for all i from 1 to n .
- 16. A number a that is relatively prime to all the integers less than $\lceil \sqrt{a} \rceil$ is prime. ($\lceil \sqrt{a} \rceil$ is literally \sqrt{x} where x is the first square larger than a .)
- 17. If one takes a fraction of a given number, and then a fraction of the remainder and continues arbitrarily, then the final remainder will be the same no matter what order the fractions were taken. Also, the sum of all the pieces taken will be the same.
- 18. If one takes a fraction of a given product, and then a fraction of the remainder, and continues arbitrarily, then the final remainder will be the same no matter in what order the fractions were taken.
- (13) 19. The number of terms in the sum $1 + 2 + \cdots + n$ is equal to the number of 1s in n .
- (14) 20. The number of odd terms in the sum $1 + 2 + \cdots + 2n$ is equal to the number of even terms.
- (15) 21. In the sum, $n + (n + 1) + (n + 2) + \cdots + (n + m)$, the last term is m greater than the first.
- (16) 22. In the sum $(n - m) + (n - m + 1) + \cdots + n + (n + 1) + \cdots + (n + m)$, the last term exceeds the middle term by the amount that the middle term exceeds the first term.
- (17) 23. In the sum $(n - m) + (n - m + 1) + \cdots + n + (n + 1) + \cdots + (n + m)$, the first term is odd iff the last term is odd.
- (18) 24. If $a - 1 = c - b$, then $a + b = c + 1$.
- (19) 25. If $a - c = c - b$ then $a + b = 2c$.
- (20) 26. $1 + 2 + \cdots + n$, where n is even, is equal to $(n/2)(n + 1)$. (Literally, half the number of terms times the number of terms plus 1. Proof works from outside in, in pairs showing that each pair sums to $n + 1$, and there are $n/2$ pairs.)
- (21) 27. $1 + 2 + \cdots + n$, where n is odd, is equal to $((n + 1)/2)n$. (Literally the middle term times the number of terms. Proof works from inside out, in pairs showing that each pair sums to twice the middle term.)

- (22) 28. $1 + 2 + \cdots + n$, where n is odd, is equal to $(n/2)(n + 1)$. (Literally, half the last term times the number after the last term. Proof uses proportions, algebralike idea, and 21).
- (23) 29. $1 + 3 + 5 + \cdots + (2n - 1) = n^2$. (Literally, the square of the middle term. Proves it first for an odd number of terms, then an even number).
- (24) 30. $(1 + 2 + \cdots + n) + (1 + 2 + \cdots + n + (n + 1)) = (n + 1)^2$.
- (25) 31. $2(1 + 2 + \cdots + n) = n^2 + n$. (Proof uses 30. Corollary: The sum $1 + 2 + \cdots + n = n^2/2 + n/2$.)
- (26) 32. $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + \cdots + n) = 2^2 + 4^2 + 6^2 + \cdots + n^2$, n even; and $1^2 + 3^2 + 5^2 + \cdots + n^2$, n odd. (Proof uses 30.)
- (27) 33. $(1 + 2 + 3 + \cdots + n) + (2 + 3 + 4 + \cdots + n) + \cdots + n = 1^2 + 2^2 + 3^2 + \cdots + n^2$. (Proof uses a counting argument.)
- (28) 34. $(1 + 2 + 3 + \cdots + n) + (2 + 3 + 4 + \cdots + n) + \cdots + n + 1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + \cdots + (n - 1)) = n(1 + 2 + 3 + \cdots + n)$. (Proof uses a counting argument.)
- (29) 35. $(n + 1)^2 + n^2 - (n + 1 + n) = 2n^2$.
- (30) 36. $(1 + 2 + 3 + \cdots + n) + (2 + 3 + 4 + \cdots + n) + \cdots + n - (1 + 2 + 3 + \cdots + n) = 2(2^2 + 4^2 + 6^2 + \cdots + (n - 1)^2)$, $n - 1$ even, and $2(1^2 + 3^2 + 5^2 + \cdots + (n - 1)^2)$, $n - 1$ odd. (Proof uses 33 and 35.)
- (31) 37. $n(1 + 2 + 3 + \cdots + (n + 1)) = 3(1^2 + 3^2 + 5^2 + \cdots + n^2)$, n odd, and $3(2^2 + 4^2 + 6^2 + \cdots + n^2)$, n even. (Proof uses 32, 34, and 36.)
- (32) 38. $(n - (1/3)(n - 1))(1 + 2 + 3 + \cdots + n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$. (Proof uses 32, 33, 34, and 37.)
- (33) 39. $(n^2 - n)/2 = (1 + 2 + \cdots + (n - 1))$. (Proof uses 30.)
- (34) 40. $(n^2 - n)/2 + n = (1 + 2 + \cdots + n)$. (Proof uses 30.)
- (35) 41. $(1 + 2 + 3 + \cdots + n)^2 = n^3 + (1 + 2 + 3 + \cdots + (n - 1))^2$. (Proof uses 30 and 6.)
- (36) 42. $(1 + 2 + 3 + \cdots + n)^2 = 1^3 + 2^3 + 3^3 + \cdots + n^3$. (Proof by induction using 41).
- (37) 43. Let $m = 1 + 2 + 3 + \cdots + n$, then $1^3 + 2^3 + 3^3 + \cdots + n^3 = 1 + 3 + 5 + \cdots + (2m - 1)$.
- (38) 44. $ab + a = (b + 1)a$, and $ab + b = (a + 1)b$.
- (39) 45. Given $a < b < c$, then $c(b - a) + a(c - b) = b(c - a)$.
- (40) 46. Given $2 < a < b$, then $2(a - 2)(b - 1) + b + (a - 2) + (b - a) = 2(a - 1)(b - 1)$. (Minor differences in the two editions.)
- (41) 47. Given $a < b$, then $ba + (b - a) = (a - 1)(b - 1) + b + (b - 1)$.
- (42) 48. Given $2 < a < b$, then $2(b - 1)(a - 2) + b + (a - 2) + (b - a) = (a - 1)b + (a - 2)(b - 1) + (b - a)$. (Minor differences in the two editions.)
- (43) 49. Given $a < b < c$, and $d = a - 2$, then $2(c - 1)(b - a) + cd + (c - 1)d + c + (b - a) + (c - b) = 2(b - 1)(c - 1)$. (Minor differences in the two editions. Generalizes 46.)
- (44) 50. Given $a < b < c$, and $d = a - 2$, then $2(c - 1)(b - a) + (cb)d + (c - b)(a - 1) + c + (b - a) = (b - 1)(c - 1)a$. (Major differences in the two editions.)

- (45) 51. Given $a < b < c$, then $(c - 1)(b - a) + c + (b - a) = c(b - a + 1)$.
- (46) 52. Given $a < b < c$, then $(c - 1)(b - a) + (a - 1)(c - b) + c + (b - a) = b(c - a + 1)$.
- (47–48) 53. Find x, y, z such that $x + (y + z)/a = y + (x + z)/b = z + (x + y)/c$, where $a < b < c \dots$.
- (49) 54. Find x , such that fractions of x minus smaller fractions of x , equals a .
- (50) 55. Given that $x + \text{big fractions of } x$ is smaller than $y + \text{small fractions of } y$, find z such that $x + \text{big fractions of } z + y$ equals $y + \text{small fractions of } z + x$.
- (51) 56. Find x such that given fractions of x equal other given fractions of a .
- (52) 57. Find x, y such that $x + \text{fractions of } y$ equals $y + \text{other fractions of } x$.
- (53) 58. Find x, y, z such that $x + z = ay$ and $y + z = bx$.
- (54) 59. $(a_1)^2(a_2)^2(a_3)^2 \dots (a_n)^2 = ((a_1)(a_2)(a_3) \dots (a_n))^2$.
- (55) $(ab^2)(ac^2) = (abc)^2$.
- (56a) 60. $(a_1)^3(a_2)^3(a_3)^3 \dots (a_n)^3 = ((a_1)(a_2)(a_3) \dots (a_n))^3$.
- (56b) $(ab^3)(a^2c^3) = (abc)^3$.
- (57) 61. $ab^2 + ba^2 = ab(a + b)$
- (58) 62. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$.
- (59) 63. $P_{n+1} = (n + 1)P_n$, where P_n is the number of different ways to order n elements. (Corollary: $P_n = n!$.)
- (60) 64. $P_{n,2} = n(n - 1)$, where $P_{n,m}$ is the number of ways to order m elements out of n .
- (61) 65. $P_{n,m+1} = P_{n,m}(n - m)$. (Corollary: $P_{n,m} = n!/(n - m)! = n(n - 1)(n - 2) \dots (n - (m + 1))$).
- (62) 66. $P_{n,m} = C_{n,m}P_m$, where $C_{n,m}$ is the number of ways to choose m elements out of n without regard to order.
- (63) 67. $C_{n,m} = P_{n,m}/P_m$.
- (64) 68. $C_{n,n-m} = C_{n,m}$.

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